Fundamental Maths Toolkit

Handbook 2020-21

Tes Institute School Direct Programme

Fundamental Mathematics Toolkit Contents

Tes Institute School Direct Programme

Fundamental Mathematics Toolkit Introduction

Since the 1st April 2020 it has become the responsibility of the ITT provider to assure trainees' English and mathematics skills. This approach replaces the professional Skills Test that previously needed to be passed prior to gaining QTS. This toolkit has been collated to support trainees to reach the required standard in mathematics to demonstrate their competence in its use to effectively carry out their role as a teacher.

Providers must assure that trainees demonstrate competence in the following areas of mathematics.

> Teachers should use data and graphs to interpret information, identify patterns and trends and draw appropriate conclusions. They need to interpret pupil data and understand statistics and graphs in the news, academic reports and relevant papers. Teachers should be able to complete mathematical calculations fluently with whole numbers, fractions, decimals and percentages. They should be able to solve mathematical problems using a variety of methods and approaches including: estimating and rounding, sense checking answers, breaking down problems into simpler steps and explaining and justifying answers using appropriate language.

Any work to address shortfalls in these skills, must be undertaken by the trainee teacher in addition to other aspects of their training and this is where the toolkit will support you. **It is the trainee's responsibility to secure Fundamental mathematics, whereas responsibility for assurance lies with the provider**.

Fundamental Mathematics Toolkit structure

Using the areas outlined by the DfE for intellectual and academic capabilities with mathematics as detailed above this has been broken down into key skills that you need to be competent in the use of. These skills have been mapped to show where you would expect to use them in the day to day life of your teaching responsibilities and it is key that these skills can be applied in context to enable you to effectively carry out all roles and responsibilities in teaching.

The toolkit provides a self-assessment grid that allows you to record your areas of strength and identify any aspects that additional action needs to be taken. This will be useful to share with your mentor and Pathway Tutor to enable them to assure your competence as well as identify targets to support your development in these areas. To support you in doing this, you will see that each skill has the following structure to enable both your ability to effectively self-assess and also develop your competence if you feel more work is needed on a specific aspect.

- An overview of the skill
- Worked examples in context to exemplify the skill
- Notes on how to avoid common errors
- Practice questions with full worked solutions so you can assess your confidence including additional support notes
- Links to free online resources that you could use as a start point to develop the aspect, if you feel further action needs to be taken

The toolkit also provides a practice paper for you to try in preparation for demonstrating your competence across the areas outlined above.

Fundamental Mathematics Skills

The guidance states:

Teachers should use data and graphs to interpret information, identify patterns and trends and draw appropriate conclusions. They need to interpret pupil data and understand statistics and graphs in the news, academic reports and relevant papers. Teachers should be able to complete mathematical calculations fluently with whole numbers, fractions, decimals and percentages. They should be able to solve mathematical problems using a variety of methods and approaches including: estimating and rounding, sense checking answers, breaking down problems into simpler steps, and explaining and justifying answers using appropriate language.

The full document can be accessed at https://www.gov.uk/government/publications/initial-teacher-training-criteria/initial-teacher-training-itt-criteria-and-supporting-advice#c13-suitability

Toolkit methodology

The toolkit provides resources that encourage independent study to develop and secure the key skills you are responsible for demonstrating. Guidance and supoprt has been taken and adapted from previous DfE guidance and support documents that were available for the Professional Skills Test as well as additional material to ensure that all aspects of the requirements that need to be demonstrated and assured are included.

The files that were used to support the creation of the toolkit are:

Areas of numeracy (DfE 2014)

Numeracy practice papers 2,3,4 (DfE 2015)

Numeracy glossary (DfE 2014)

All can be accessed at

https://drive.google.com/drive/folders/1uVVwwo1OuRIKTIkpjt-JISE7YnVe4ICq?usp=sharing

Toolkit Skills Section

Fundamental Maths Skills to be Demonstrated

1 Non Calculator Skills	**Possible Teacher use**
1.1 Calculate with whole numbers and decimals	Working out lesson timings or durations.
1.2 Find simple percentages or fraction of a number such as 10%, 25% or ¾ of 20	Deciding the amount of printing needed for different worksheets
1.3 Use rounding to estimate and check if answers are sensible	For example, to check the reasonableness of the calculation of the cost of an order of multiple resources, including delivery. To quickly establish the financial viability of a school trip or visit
2 Mathematical calculations and problem solving	**Possible Teacher use**
2.1 Convert between numbers expressed as a fraction, a decimal and as a percentage	Comparing quantities expressed in different forms. To allow comparisons. For example, choose between offers such as '25% reduction' or '1/3 off' when purchasing resources in a sale.
2.2 Use a calculator to find a percentage of a quantity such as 17.5% of £50	Forecasting student progress
2.3 Calculate a percentage increase or decrease	For example, calculate the additional cost of a trip per pupil when a coach company increases its prices.
2.4 Use percentages to compare data	For example, calculate and compare a pupil's scores from two tests with different numbers of marks. Calculate percentage scores from a summative assessment
2.5 Use ratio and proportional reasoning to solve problems	For example, to find the best buy, conversion between miles and kilometres or adapt a recipe for more or less people.
2.6 Simplify ratios	Comparing teacher pupil ratios across schools.
2.7 Solve mathematical problems by breaking them down into a series of simpler steps and selecting appropriate operations (+,-,÷,x) or using simple formulae	Calculating the cost of an order of multiple resources, including delivery. To calculate costs of a school trip or visit. Calculating the overall grade for an exam with unequally weighted papers.
3.Data interpretation	**Teacher use**
3.1 Summarise and compare data sets by finding the mean, mode, median, interquartile range and range	Use data from research papers to inform teaching. Comparing class performance against targets.
3.2 Interpret and draw conclusions from data presented in tables, such as a two way table, a bar chart or as a pie chart	Interpret graphs in media articles relevant to a curriculum area or those related to education
3.3 Interpret and draw conclusions from data relationships presented in a scatter diagram	In deciding if an overall grade can be estimated for a student that missed one of two exams.
3.4 Compare and draw conclusions from two or more data sets presented graphically for example as a box plot or a cumulative frequency diagram	Understanding school performance data
3.5 Identify patterns or trends within data sets presented graphically such as a line graph or a time series graph	Interpret graphs in media articles relevant to a curriculum area or those related to education. Understand the trends in school performance data.

Toolkit Skills Audit

Rate your competence against each skill

1 Non Calculator Skills	🟥	🟧	🟩
1.1 Calculate with whole numbers and decimals			
1.2 Find simple percentages or fraction of a number such as 10%, 25% or ¾ of 20			
1.3 Use rounding to estimate and check if answers are sensible			
2 Mathematical calculations and problem solving			
2.1 Convert between numbers expressed as a fraction, a decimal and as a percentage			
2.2 Use a calculator to find a percentage of a quantity such as 17.5% of £50			
2.3 Calculate a percentage increase or decrease			
2.4 Use percentages to compare data			
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3.5 Identify patterns or trends within data sets presented graphically such as a line graph or a time series graph.			

1.1 Calculate with whole numbers and decimals

Non-Calculator skills

There are many methods to accurately and confidently carry out calculations without formal written methods. Knowing a few of these will help.

Below are some of the questions from the Non-Calculator section from the Numeracy Practice Test 2.

The questions have been answered to illustrate some of the methods that are useful to know. Formal written methods should not be necessary for these questions. However, being able to do long multiplication and division will be an advantage when a calculator is not available.

Question 1

Teachers organised activities for three classes of 24 pupils and four classes of 28 pupils.

What was the total number of pupils involved?

Calculation required. $3\times24 + 4\times28$

method 1:

$3\times(25-1) = 3\times25-3\times1$

$= 75-3 = 72$

$4\times(25+3) = 4\times25+4\times3$

$= 100+12 = 112$

$$\begin{array}{r} 112 \\ +\ 72 \\ \hline 184 \end{array}$$

method 2:

add 3 ones to the 24's from a 28.

calculation is now $4\times25 + 3\times28$

$100 + 3\times(25+3)$

$100 + 75+9 = 184$

Recognising that 28 and 24 are near 25 is useful. Look out for near multiples you know.

method 3:

$$\begin{array}{r} 24 \\ \times\ 3 \\ \hline 72 \\ \hline {}_{1} \end{array} \qquad \begin{array}{r} 28 \\ \times 4 \\ \hline 112 \\ \hline {}_{3} \end{array} \qquad \begin{array}{r} 112 \\ +\ 72 \\ \hline 184 \end{array}$$

method 4:

$7\times25 = 175$

and adjust. $175-3\times1+4\times3$

$175-3+12 = 184$

methods 1, 2, and 4 are possible with little or no writing.

Question 2

All 30 pupils in a class took part in a sponsored spell to raise money for charity. The pupils were expected to get an average of 18 spellings correct each. The average amount of sponsorship was 20p for each correct spelling.

How many pounds would the class expect to raise for charity?

Calculation required. 30 × 18 × 20p or 30 × 18 × £0·20

method 1:

30 × 18 × 0·2
= 30 × 0·2 × 18
(÷10) (×10)
= 3 × 2 × 18
= 6 × 18
= (5+1) × 18
= 5 × 18 + 18
= 90 + 18
= £108

When multiplying changing the order may help and does not change the answer

Multiplications can be adjusted to make them easier. Multiply a number by something and divide another by the same to compensate. The answers will be the same!

method 2:

18 × 30 = 18×3×10
= 54×10
= 540

Breaking down a number will allow more changes to the order to multiply.

540 × 20 = 54 × 10 × 2×10
= 54 × 2 × 10×10
= 108 × 100
= 10800 p
÷100
= £108

Could have missed these steps and just adjusted the units.

method 3:

30 × 18 × 20
= 3×10 × 2×9 × 2×10
= 3×2×9×2 × $\underline{10 \times 10}$
= 6 × 2 × 9 × 100
= 12 × 9 × 100
= 108 × 100p
= £108

method 4:

30 × 18 × 20

$$\begin{array}{r} 30 \\ \times\ 18 \\ \hline 240 \\ 300 \\ \hline 540 \end{array} \qquad \begin{array}{r} 540 \\ \times\ 20 \\ \hline 0 \\ 10800 \\ \hline \end{array}$$

10800p = £108

Method 4 may seem the safest one to use, but others with practice will become quicker and maybe more accurate. When working through these and later questions, try to find as many ways as possible to answer them.

Question 3

As part of the numeracy work in a lesson, pupils were asked to stretch a spring to extend its length by forty per cent.
The original length of the spring was 45 centimetres.

What should be the length of the extended spring?
Give your answer in centimetres.

Calculation required $45 + 40\%$ of 45

method 1:

$10\% = 4.5$ $(\div 10)$

×2 ×2

$20\% = 9$

×2 ×2

$40\% = 18$

$$\begin{array}{r} 45 \\ +\ 18 \\ \hline 63\text{cm} \\ \hline {}_1 \end{array}$$

method 2:

$45 + \underset{(\times 10)}{0.4} \times \underset{(\div 10)}{45}$

$= 45 + \underset{(\div 2)}{4} \times \underset{(\times 2)}{4.5}$

$= 45 \times 2 \times 9$

$= 45 + 18$

$= 63$cm

method 3:

$45 + 40\%$ of $45 = 140\%$ of 45

140% of 45

$= \underset{(\times 10)}{1.4} \times \underset{(\div 10)}{45}$

$= \underset{(\div 2)}{14} \times \underset{(\times 2)}{4.5}$

$= 7 \times 9$

$= 63$cm

Adding the percentage increase to 100% before finding it may make it easier than adding it after.

Question 4

For a science experiment a teacher needed 95 cubic centimetres of vinegar for each pupil. There were 20 pupils in the class.
Vinegar comes in 1000 cubic centimetre bottles.

How many bottles of vinegar were needed?

Calculation needed $95 \times 20 \div 1000$ then round up to nearest whole number.

method 1:

$95 \times 20 \div 1000$

$\approx 100 \times 20 \div 1000$

$2000 \div 1000$

$= 2$

Sometimes rounding before calculating will produce an accurate enough answer

method 2:

$95 \times 20 = (100 - 5) \times 20$

$= 100 \times 20 - 5 \times 20$

$= 2000 - 100$

$= 1900$

$1900 \div 1000 = 1.9$

So need 2 bottles

method 3:

$$\begin{array}{r} 95 \\ \times\ 20 \\ \hline 0 \\ 1900 \\ \hline 1900 \\ \hline \end{array}$$

$1900 \div 1000$

$= 1.9$

need 2 bottles

method 4:

means approximately equal to.

$1000 \div 95 \approx 1000 \div 100 = 10$

So one bottle will be enough for 10 pupils.

2 bottles needed for 20.

Question 5

The morning session in a school began at 09:25.

There were three lessons of 50 minutes each and one break of 20 minutes.

At what time did the morning session end?

Give your answer using the 24-hour clock.

Calculation needed add 3 x 50 + 20 minutes to 09:25

method 1:

3 x 50 = 150

150 + 20 = 170

170 ÷ 60 = 2 r 50

2 hours 50 minutes

add 2 hours 50 mins to 09:25

09:25 + 3 hours − 10 mins

= 12:25 − 10 mins

= 12:15

method 2:

09:25 + 2 hours 50 mins

+35mins +2h +15mins

09.25 10·00 12·00 12.15

split the 50 mins to up to 10·00 am and after 2 hours later.

See the time notes below for more questions and examples.

Question 6

What is six hundred and forty-three divided by zero point one?

.

Calculation needed

$643 \div 0.1$

method 1

When adjusting division calculations to make them easier a different method is needed than for multiplication. This time you need to multiply or divide both numbers by the same number.

$\underset{\times 10}{643} \div \underset{\times 10}{0.1}$

$= 6430 \div 1 = 6430$

method 2:

$643 \div 0.1 = 643 \div \frac{1}{10}$

$= 643 \times \frac{10}{1}$

when dividing by a fraction flip it and multiply.

$= 643 \times 10$

$= 6430$

Question 7

A teacher took a group of pupils to an aquarium whilst visiting France.

The total entrance cost for the group was 160 euros.

Taking 1.6 euros as equal to one pound, what was the total entrance cost, in pounds, for the group of pupils?

Calculation needed $160 \div 1.6$

Method 1:

$160 \div 1.6$

$(\times 10) \qquad (\times 10)$

$1600 \div 16$

$(\div 16) \qquad (\div 16)$

$100 \div 1 = 100$

Method 2:

$$160 \div 1.6$$

$$= 160 \div 1\tfrac{6}{10}$$

$$= 160 \div \frac{16}{10}$$

$$= 160 \times \frac{10}{16}$$

$$= \frac{160 \times 10}{16} = \frac{1600}{16} = 100$$

Method 3:

$$\begin{tikzcd}
\text{Euros} & : & \text{£} \\
1.6 \arrow[d, bend right, "\times 100"'] & & 1 \arrow[d, bend left, "\times 100"] \\
160 & & ?
\end{tikzcd}$$

$1 \times 100 = £100$

Using proportional reasoning can be good for exchange rate questions. See section 2.5 for more detail

Question 8

A school's policy for Key Stage 2 was to set three and a half hours of homework per week.

What was the mean number of minutes to be spent on homework per weekday evening?

Calculation needed. Convert $3\frac{1}{2}$ hours into minutes and divide by 5

method 1:

$$3\tfrac{1}{2} \times 60 = 3\times 60 + \tfrac{1}{2}\times 60$$
$$= 180 + 30$$
$$= 210$$

$$\underset{\times 2}{210} \div \underset{\times 2}{5}$$
$$= 420 \div 10$$
$$= 42 \text{ minutes}$$

method 2:

$$210 \div 5 = 210 \div (10\times 2)$$
$$= 21 \times 2$$
$$= 42 \text{ minutes}$$

Time

In order to calculate answers to questions involving time, you must know which units are being used. In questions referring to a week, please note that it is a school week of 5 days.

When working out children's reading ages it is necessary to consider time given in years and months. A child may have an actual age of 8 years and 4 months, but a reading age of 8 years and 10 months. Another child of the same age might have a reading age of 7 years and 9 months. Reading ages are presented in several forms and 8 years 6 months, 8.6 or 8-6 are all common. If the form 8.6 is used, it has to be remembered that here the point is not a decimal point, just a means of separating months and years.

Example 1

A junior school is putting on entertainment. If each of the 4 classes is allowed 25 minutes and the entertainment starts at 2.15pm, when is it expected to finish?

If each class were allowed half an hour, the entertainment would finish 2 hours after the start, therefore 4.15pm.

However, each class has 25 minutes, which is 5 minutes less than half an hour. So 4 classes would take 4 x 5 minutes less, ie 20 minutes less.

The entertainment should therefore finish 20 minutes before 4.15pm. 15 minutes before 4.15pm is 4.00pm, 20 minutes before 4.15pm is 3.55pm.

An alternative method would be to calculate:

25 minutes x 4 = 100 minutes = 1 hour 40 minutes

So the 4 classes take 1 hour 40 minutes.

1 hour 40 minutes after 2.15pm is 3.55pm.

Example 2

A parent comes in to a school for 1 and a half hours to hear a group of children reading. If there are 10 children in the class, can she give them all 10 minutes each?

One and a half hours is 60 + 30 = 90 minutes.

To give 10 children 10 minutes each would take 100 minutes. So she cannot give each of them 10 minutes.

Avoiding common errors

Most common errors can be avoided by:

- checking the units used in the question;
- not using units of time as if they were decimals - for example, when referring to reading ages 7.5 does not mean 7½ years (7 years 6 months), but 7 years 5 months. To avoid confusion, the notation 7-5 is often used;
- being clear about am and pm on the 12-hour clock, and
- taking care with times between 12.00 and 24.00 on the 24-hour clock.

Now your turn 1.1

The follow questions should be answered **without a calculator**. Pen and paper can be used.

1.1a

During a school trip to Germany, each pupil was allowed to exchange £100 into euros for spending money.
The exchange rate was €1.06 to the pound. How many euros did each pupil receive?

1.1b

Ninety pupils travelled to an exhibition on two coaches. Each coach cost £180 to hire.
The total entrance fee to the exhibition for all pupils was £90. How much did each pupil have to pay to meet the total cost?

1.1c

The school library is open for
5 hours 20 minutes per day on Monday, Wednesday and Friday,
and for 6 hours 30 minutes per day on Tuesday and Thursday.
What is the total time the library is open during the school week?

1.1d

A primary school concert cost £80 for promotion, costumes and refreshments. The parents attending each paid a £1.50 entrance fee. Sixty parents attended.

How much money was left for the school funds?

1.1e

A school sold copies of the class photograph to 28 pupils in the class for £2.50 each.

What was the total amount raised from the sales of the photograph?

1.1f

A teacher plans a cross-country competition.

The course is 3.45 kilometres long. Pupils do 3 laps of the course.

What is the total distance run by each pupil in kilometres?

1.1g

A school play begins at 19:00 hours. It has one interval of 15 minutes. The play is 1 hour and 55 minutes long.

At what time does the play end?
Give your answer using the 24-hour clock.

The answers are on the next page.

The worked solutions show only one possible method, others are equally valid.

1.1a worked solution.

During a school trip to Germany, each pupil was allowed to exchange £100 into euros for spending money.

The exchange rate was €1.06 to the pound. How many euros did each pupil receive?

1.06 x 100 = €106

1.1b

Ninety pupils travelled to an exhibition on two coaches. Each coach cost £180 to hire.

The total entrance fee to the exhibition for all pupils was £90. How much did each pupil have to pay to meet the total cost?

Coach hire(2x180) ÷ 90 =£360÷90= £4.00 each

Entrance 90÷90 = £1 each

total cost per student = 4+1 = £5.00

1.1c

The school library is open for

5 hours 20 minutes per day on Monday, Wednesday and Friday,

and for 6 hours 30 minutes per day on Tuesday and Thursday.

What is the total time the library is open during the school week?

5 hr 20 × 3 + 6 hr 30 × 2

5×3 = 15 hrs 6 ×2 = 12 hours

20 × 3 = 60 mins = 1 hr 30×2 = 60 mins = 1 hour.

total = 15 + 12 + 1 + 1

= 29 hours per week

1.1d

A primary school concert cost £80 for promotion, costumes and refreshments. The parents attending each paid a £1.50 entrance fee. Sixty parents attended.

How much money was left for the school funds?

60 × 1·50 − 80

60 × (1 + 0·5) − 80

60 + 30 − 80

90 − 80 = £10 for the school fund

1.1e

A school sold copies of the class photograph to 28 pupils in the class for £2.50 each.

What was the total amount raised from the sales of the photograph?

28×2.50

$= 14 \times 5$

$= 7 \times 10$

$= £70$

1.1f

A teacher plans a cross-country competition.

The course is 3.45 kilometres long. Pupils do 3 laps of the course.

What is the total distance run by each pupil in kilometres?

3.45×3

$= 3.45 \times (2 + 1)$

$= 3.45 \times 2 + 3.45 \times 1$

$$\begin{array}{rr} = & 6.90 \\ + & 3.45 \\ \hline & 10.35 \text{ km} \end{array}$$

1.1g

A school play begins at 19:00 hours. It has one interval of 15 minutes. The play is 1 hour and 55 minutes long.

At what time does the play end?
Give your answer using the 24-hour clock.

1 hour 55 mins + 15 mins = 2 hrs 10 mins

19.00 + 2hrs 10 mins = 21:10

1.1 Further learning and support

See chapters 2,4,6,8 and 12. http://www.cimt.org.uk/projects/mepres/book7/book7int.htm
Mental arithmetic resources http://mrbartonmaths.com/topics/number-skills/mental-arithmetic/
Written arithmetic resources http://mrbartonmaths.com/topics/number-skills/written-arithmetic/
Decimal resources http://mrbartonmaths.com/topics/number-skills/place-value/
Time resources see chapter 14 http://www.cimt.org.uk/projects/mepres/book7/book7int.htm#unit14

1.2 Find simple percentages or fraction of a number such as 10%, 25% or ¾ of 20

Simple percentage calculations

Some percentages are very easy to calculate both in your head and on a calculator. For example, it is always straightforward to find 10% of a quantity in your head by remembering that 10% is $\frac{1}{10}$. So, to find 10% of anything, all you need to do is divide by 10.

What is 10% of 632?

10% of 632 = 632 ÷ 10 = 63.2

Finding 10% can often help with finding other percentages in your head.

Examples

1. What is 30 % of 40?
   10% of 40 = 40 ÷ 10 = 4
   30% of 40 = 4 x 3 = 12

2. What is 15 % of 30?
   10% of 30 = 3
   5% of 30 = 3 ÷ 2 = 1.5
   15% of 30 = 3 + 1.5 = 4.5

The other common percentages can be used in the same way to find answers in your head.
50% divide by 2 as 2 x 50 =100
25% divide by 4 as 4 x 25 = 100
75% find 25% and times by three as 3 x 25 =75
20% divide by 5 as 5 x 20 = 100 or find 10% and double answer.

Simple fraction calculations

Some fractions of a quantity are very easy to calculate both in your head and on a calculator.

Divide the quantity by the fraction's denominator (bottom number) and the multiple by the numerator (top number).

For example

(a) Find $\frac{1}{5}$ of £30.

(b) Find $\frac{4}{5}$ of £30

a) £30 ÷ 5 = £6
£6 x 1 = £6

b) £30 ÷ 5 = £6
£6 x 4 = £24

Now your turn 1.2a

A school trip was planned at a total cost of £120 per pupil. The accommodation cost two-fifths of the total.

What was the cost of the accommodation per pupil?

Now your turn 1.2b

In a year group of 120 pupils, 75% were working at or above the expected level in English. 65 pupils were working at the expected level.

How many pupils were working above expected level?

The answers are on the next page.

Now your turn 1.2a worked solution

A school trip was planned at a total cost of £120 per pupil. The accommodation cost two-fifths of the total.

What was the cost of the accommodation per pupil?

$\frac{2}{5}$ of 120

120 ÷ 5 = 24

24 × 2 = 48

to divide by 5, divide by 10 double the answer

£48 per pupil

Now your turn 1.2b worked solution

In a year group of 120 pupils, 75% were working at or above the expected level in English. 65 pupils were working at the expected level.

There are many ways to find percentages such as 75%.
100% − 25%
or 7 × 10% + 5% (half of 10%)
When working mentally explore different methods

How many pupils were working above expected level?

75% of 120 is the same as $\frac{3}{4}$ of 120.

120 ÷ 4 = 30   30 × 3 = 90   90 − 65 = 25 working above

÷ by 2 twice to ÷ by 4

1.2 Further learning and support.

Fractions of an amount https://www.mathsgenie.co.uk/fraction-of-amount.html

Percentages https://www.mathsgenie.co.uk/percentages.html or https://corbettmaths.com/2012/08/20/percentages-of-amounts-non-calculator/

1.3 Use rounding to estimate and check if answers are sensible

Rounding

Numbers can be rounded when all that is required is a *reasonable approximation* rather than the exact value. There are two ways to round numbers, by considering the number of decimal places or by using significant figures. For the purpose of estimation, it is usual to round numbers to one significant figure. This has the benefit of usually making the calculation simple enough to be done mentally but accurate enough to check if answers are reasonable.

Significant figures

When rounding using significant figures attention is given to the relative importance of the digits of the numbers. The first non-zero digit being the most significant or important.

Examples

Write:

1. 7.621 correct to 2 significant figures.
2. 537 981 correct to 3 significant figures.
3. 0.038 correct to 1 significant figure.

1. 7.6 correct to 2 significant figures. Note the third significant figure (2) is used in the rounding process. As it is **below 5** the first two digits remain **unchanged**.
2. 538 000 correct to 3 significant figures (3 sf). Note that in this case the fourth significant figure (9) is used in the rounding process. As it is **5 or greater** the 3rd digit is **rounded up** to 8 from 7. Also note that three zeros are needed to replace the reminding digits of the original number, to give a rounded number of a comparable size.
3. 0.04 correct to 1 sf. Note that the zeros at the beginning have not been used as they are not significant, it's the 3, the first significant figure that really determines the numbers size. If three is the first then the second significant figure (8) is used in the rounding process. As 8 is **5 or greater** the 3 is **rounded up** to 4. The zeros at the beginning are still required, to give a number of a comparable size.

Estimation

We can estimate the answers to calculations by rounding all the numbers sensibly. We often round to just one significant figure (1sf). However, depending on the numbers involved in the calculations, it may better to round sensibly rather than to 1sf to produce a close estimation. This is more common when the calculation involves division, rounding to numbers within known times tables.

For example:

Estimate 45.47 ÷15. 36 If rounding to 1sf, the estimation would be 50 ÷ 20, which is not easily to do mentally. However, if rounded sensibly, the estimation would be **45 ÷ 15 = 3**. This is both easier and more accurate as the numbers have been rounded less. Knowing that 3 x 15 = 45 was useful.

Worked examples

Example 1.

A box of pens costs £3.72. Estimate the cost of 6 boxes.

Cost (£) = 6 x 3.72

Estimate = 6 x 4

= £24

Example 2.

A school minibus uses 0.15 litres of petrol per mile. Estimate the amount of fuel needed for a journey of 246 miles.

Amount (litres)= 246 x 0.15

Estimate = 200 x 0.2

= 40 litres

Example 3.

A teacher used a calculator to work out 997.368 ÷ 12.4

The answer on the calculator was 788.2. Use estimation to check the answer.

Calculation = 997.368 ÷ 12.4

Estimate = 1000 ÷ 10

= 100

The answer should have been near 100, so it is likely the decimal point has been misplaced, the answer possibly being 78.82. The teacher should re do the calculation.

Example 4.

Estimate 73% of 84

Calculation = 0.73 x 84

Estimate = 0.75 x 80

= 3/4 of 80 = 60

Avoiding common errors

Most errors can be avoided by:

- Correctly identifying the first significant figure.
- Replacing digits with zeros to ensure the rounded number is reasonable.
- Using sensible rounding if using one significant figure produces a difficult calculation.

Now your turn 1.3

A teacher is planning a trip to the Tate Modern. Entrance is free, but the petrol for the minibus will need to paid for by the 13 students in the art class. The London congestion charge will add £1 per student. The school estimates that the minibus uses 0.089 litres of petrol per mile. The cost of petrol is 114.9p per litre. The teacher used Google Maps to calculate that the return journey would be about 136 miles.

According to their calculations, each student will need to pay £3.13. Use estimation to check if this could be correct.

The answer is on the next page.

Now your turn 1.3 worked solution.

A teacher is planning a trip to the Tate Modern. Entrance is free, but the petrol for the minibus will need to paid for by the 13 students in the art class. The London congestion charge will add £1 per student. The school estimates that the minibus uses 0.089 litres of petrol per mile. The cost of petrol is 114.9p per litre. The teacher used Google Maps to calculate that the return journey would be about 136 miles.

According to their calculations, each student will need to pay £3.13. Use estimation to check if this could be correct.

Calculation: $0{\cdot}089 \times 136$ = number of litres required.

$\times\ 114{\cdot}9$ = cost of fuel.

total cost of fuel $\div\ 13$ = fuel cost per student (pence)

$+$ £1 congestion charge.

$$(0{\cdot}089 \times 136 \times 114{\cdot}9)_p \div 13 + £1{\cdot}00$$

Estimation $= (0{\cdot}09 \times 100 \times 100)p \div 10 + £1{\cdot}00$

(×100) (÷100)

$= (9 \times 1 \times 100)p \div 10 + £1{\cdot}00$

$= 900p \div 10 + £1{\cdot}00$

$= 90p + £1{\cdot}00$

$= £1{\cdot}90$

Estimations should ideally be done <u>before</u> the calculation it is checking.

The answer of £3.13 appears too high. The teacher should re check the calculation.

(note the £3·13 was calculated wrongly as the teacher used double the distance)
Correct cost is £2·07 per student.

1.3 Further learning and support.

https://www.mathsgenie.co.uk/estimating.html

https://corbettmaths.com/2012/08/21/approximation-to-calculations/

2.1 Convert between numbers expressed as a fraction, a decimal and as a percentage

Fractions, decimals and percentages

Fractions, decimals and percentages are all forms for expressing parts of a whole and each can be represented in any of the 3 forms. For example, the fraction ½ can be represented as a decimal (0.5) or as a percentage (50 %).

Converting fractions to decimals and vice versa

Converting a fraction to a decimal

A fraction can always be converted to a decimal by dividing the top (numerator) by the bottom (denominator).

So the fraction $^{60}/_{150}$ can be converted to a decimal by division: 60 ÷ 150, or 6 ÷ 15, or 2 ÷ 5.

Each of these will give the same answer 0.4, because all 3 fractions have the same value, that is, they are equivalent.

It is worthwhile remembering some common equivalents, this will support the Non Calculator Fundamental Maths Skills (1.1 to 1.3)

$^{3}/_{4} = 0.75$

$^{1}/_{2} = 0.5$

$^{1}/_{4} = 0.25$

Converting a decimal to a fraction

A decimal such as 0.7 can be written as a fraction by remembering that the first decimal represents tenths, the second hundredths and so on.

So 0.7 = $^{7}/_{10}$, 0.46 = $^{46}/_{100}$ and so on.

Sometimes the result of division gives more digits after the decimal point than are needed or are appropriate. In this case the number would be rounded up or down, remembering always to state the degree of accuracy that has been given. So an answer 63.472418 might be rounded up to 63.5 to 1 decimal place or to 63.47 to two decimal places.

Converting fractions and decimals to percentages and vice versa

Converting a percentage to a fraction

As a percentage represents parts out of a hundred, it is easy to convert a percentage to a fraction.

So 60% = $^{60}/_{100}$
= $^{30}/_{50}$ (by dividing top and bottom numbers by 2)
= ⅗ (by dividing top and bottom numbers by 10)

75% = $^{75}/_{100}$
= ¾ (by dividing top and bottom numbers by 25)

If using a scientific calculator, simply key in the percentage and divide by 100, press the equals button. The calculator will return the simplified fraction. Most scientific calculators also have a S<>D button. Pressing this will toggle the answer between being expressed as a decimal and a fraction.

Converting a decimal to a percentage

It is easy to convert a decimal to a percentage, because, as already seen above, decimals can always be easily written with a denominator of a hundred.

0.5 = ½
= $^{50}/_{100}$
= 50%

0.37 = $^{37}/_{100}$
= 37%

0.413 = $^{413}/_{1000}$
= $^{41.3}/_{100}$
= 41.3%

Converting a fraction to a percentage

Fractions with denominators of 10 or closely related to 10 can be converted to a percentage as follows.

$^{7}/_{10} = {}^{70}/_{100} = 70\%$

$^{3}/_{5} = {}^{6}/_{10} = {}^{60}/_{100} = 60\%$

$^{9}/_{20} = {}^{45}/_{100} = 45\%$

$^{7}/_{25} = {}^{28}/_{100} = 28\%$

Example

A pupil scores $^{71}/_{80}$ in a test; what percentage is this?

71÷ 80 using a calculator gives 88.75%.

For most fractions it is easier to convert them first into decimals (using a calculator if appropriate) and then convert that decimal into a percentage.

$^{4}/_{9}$ = 4 ÷ 9 = 0.444444 = 44.4444% = 44% to the nearest whole number.

$^{13}/_{17}$ = 13 ÷ 17 = 0.7647058 = 76.47058% = 76% to the nearest whole number, or 76.5% to 1 decimal place.

Some of the most common fractions, decimals and their percentage

equivalents: 1% = 1⁄100 = 0.01 (divide by 100)

5% = 1⁄20 = 0.05 (divide by 20)

10% = 1⁄10 = 0.1 (divide by 10)

12.5% = 1⁄8 = 0.125 (divide by 8)

20% = 1⁄5 = 0.2 (divide by 5)

25% = 1⁄4 = 0.25 (divide by 4)

50% = 1⁄2 = 0.5 (divide by 2)

75% = 3⁄4 = 0.75 (divide by 4, multiply by 3)

You should learn all of these and be able to switch between equivalent forms. This knowledge could be particularly helpful when answering questions from skill 1.2

Examples

1. A maths department ordered books to the value of £96. The supplier gave a discount of 12.5%. ⅛ x 96 =12

   The saving is £12, so the books actually cost £84.

2. Three eighths of a class of 30 pupils have school dinners. What percentage do not have school dinners?

   ⅜ = 37.5%, so 62.5% do not have school dinners.

Now your turn 2.1a

Six out of 28 pupils scored full marks in a test. What percentage of pupils scored full marks? Give your answer correct to 1 decimal place.

Now your turn 2.1b

Three teachers recorded average class marks as follows:

Class A	35%
Class B	9 out of 20
Class C	0.4

Determine the order of the averages, starting with the best performing class.

The answers are on the next page.

Now your turn 2.1a worked solution.

Six out of 28 pupils scored full marks in a test. What percentage of pupils scored full marks? Give your answer correct to 1 decimal place.

$6 \div 28 = 0.21428...$

$0.21428... \times 100 = 21.428...$

$= 21.4\%$

On a scientific calculator $6 \div 28 = \frac{3}{14}$

Pressing the S⇔D button converts it to 0.21428...

Now your turn 2.1b worked solution.

Three teachers recorded average class marks as follows:

Class A 35% ③

Class B 9 out of 20 $9 \div 20 \times 100 = 45\%$ ①

OR non calculator $\frac{9}{20}\frac{\times 5}{\times 5} = \frac{45}{100} = 45\%$.

Class C 0.4 $\times 100 = 40\%$ ②

Determine the order of these averages, starting with the best performing class.

class B, class C, class A

$\frac{9}{20}$, 0.4, 35%

To compare, the averages need to be in the same form. Percentages are the easiest to work with.

2.1 Further learning and support.

Converting https://corbettmaths.com/2013/02/15/fdp/

Ordering https://corbettmaths.com/2013/04/15/ordering-fractions-decimals-percentages/

All things decimals and fractions https://www.cimt.org.uk/ske/A1/gcsesupport.htm

2.2 Use a calculator to find a percentage of quantity such as 17.5% of £50

Other more complex percentages can also be found easily by converting the percentages to a decimal, using a calculator if appropriate.

What is 73% of 84?

Convert 73% into a decimal in your head

or by dividing by 100 on the calculator.

73% = 0.73

0.73 x 84 on the calculator, or by other means = 61.32

You can **check** whether your answer is about right by estimation. For example, 73% is about 75% which you know is ¾. So, you know the answer will certainly be more than ½ of 84 but less than 84.

Now your turn 2.2

As part of a data handling exercise, a group of pupils was investigating attendance at after-school clubs at their school.
A survey of pupils shows that at least 30% of all the 256 pupils in the school want to come to after-school clubs.
On a particular day, the attendance was as shown in the table.

Attendance at after-school clubs	
Chess	5
Computing	15
Drama	8
Music	7
Sports	18
Total	**53**

What is the minimum number of extra pupils, above the 53, who want to come to after-school clubs?

The answer is on the next page.

Now your turn 2.2 worked solution

As part of a data handling exercise, a group of pupils was investigating attendance at after-school clubs at their school.

A survey of pupils shows that at least 30% of all the 256 pupils in the school want to come to after-school clubs.

On a particular day, the attendance was as shown in the table.

Attendance at after-school clubs	
Chess	5
Computing	15
Drama	8
Music	7
Sports	18
Total	**53**

What is the minimum number of extra pupils, above the 53, who want to come to after-school clubs?

30% of 256 = $0 \cdot 3 \times 256 = 76 \cdot 8$, so at least 77 pupils want to come to after school clubs.

rounded up as want at least 30%. Rounding down would be below 30%.

$77 - 53 = 24$ extra

2.2 Further learning and support.

http://www.cimt.org.uk/projects/mepres/book7/bk7i17/bk7_17i3.htm
And https://corbettmaths.com/2013/02/15/percentages-of-an-amount-calculator/

2.3 Calculate a percentage increase or decrease

Percentage changes

Often prices and other amounts are changed by increasing or decreasing the amount by a percentage. There are two main ways these can be calculated. The examples below illustrate these methods.

Worked examples

Example 1

Sally earns £70 a week from a part-time job. She is to be given a 6% pay rise. How much will she earn after the pay rise?

Method A	Method B
100% + 6% = 106%	Increase = 6% of £70 = 0.06 x £70
Which is 1.06 as a decimal (÷ 100)	= £4.20
New Pay = 1.06 x £70 = £74.20	New Pay = £70 + £4.20 = £74.20

Example 2

An audio company offers a 15% discount for teachers. If a speaker usually costs £179, what would a teacher expect to pay.

Method A	Method B
100%-15% = 85%	Discount = 15% of £179 = 0.15 x £179
As a decimal 0.85	= £26.85
Price for a teacher = 0.85 x £179 = £152.15	Price for a teacher = £179 - £26.85 =£152.15

Example 3

A one-year Fixed Rate Cash ISA has an AER of 0.65%. If the minimum investment of £500 was made, what would the investment be worth one year later.

Method A	Method B
100% + 0.65% = 100.65%	Interest = 0.65% of £500 = 0.0065 x £500
As a decimal 1.0065	= £3.25
Investment value = 1.0065 x £500 = £503.25	Investment value = £500 + £3.25 = £503.25

The first method makes the change to the percentage before any calculation involving the amounts are made. The second method finds the percentage of the amount then makes the change.

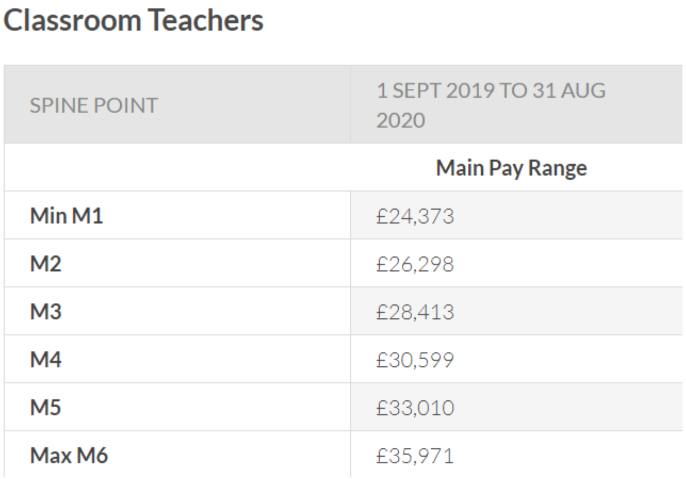
Avoiding common errors

Most errors can be avoided by:

- Carefully changing the percentage to a decimal. Dividing (small) numbers by 100 can be difficult so using a calculator is advisable.
- Checking if the percentage change requires addition or subtraction.
- When dealing with money calculations remember to give answers to the nearest penny (two decimal places), rounding if necessary.

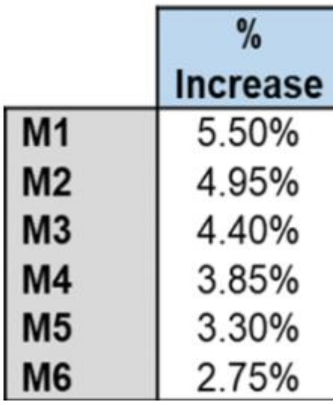
Now your turn 2.3a

The tables below show the 2019-2020 classroom teacher pay scale and the percentage increases for September 2020-2021.



Classroom Teachers

SPINE POINT	1 SEPT 2019 TO 31 AUG 2020
	Main Pay Range
Min M1	£24,373
M2	£26,298
M3	£28,413
M4	£30,599
M5	£33,010
Max M6	£35,971



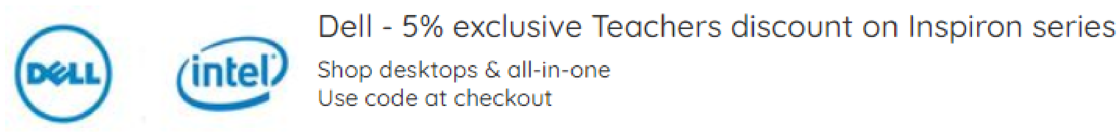
	% Increase
M1	5.50%
M2	4.95%
M3	4.40%
M4	3.85%
M5	3.30%
M6	2.75%

Sources: https://www.nasuwt.org.uk/advice/pay-pensions/pay-scales/england-pay-scales.html
And https://schoolsweek.co.uk/confirmed-5-5-pay-rise-for-new-starters-and-2-75-increase-for-all-other-teachers/#:~:text=This%20means%20the%20minimum%20starting,to%20%C2%A330%2C000%20by%202022.

A recently appointed teacher will be on paid on spine point 2 from September 2020. What will her salary be?

Now your turn 2.3b

A teacher wishes to buy a new laptop and found this advert.



Dell - 5% exclusive Teachers discount on Inspiron series

Shop desktops & all-in-one

Use code at checkout

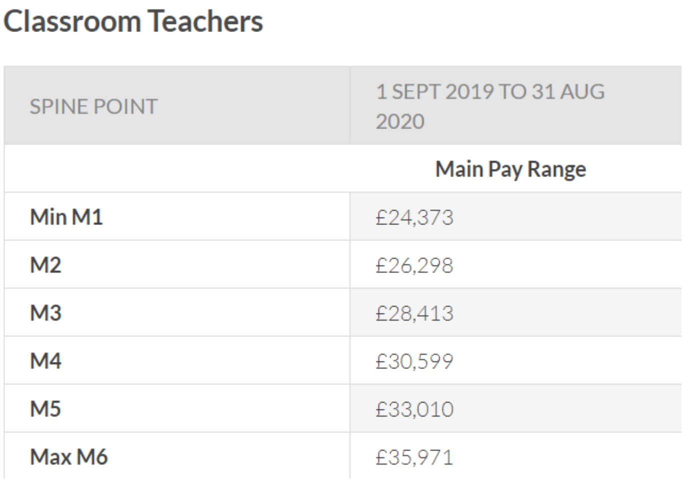
Source: https://www.discountsforteachers.co.uk/shopping/electrical/computing

What would the discounted cost be on a laptop costing £329?

The answers are on the next page.

Now your turn 2.3a worked solution.

The tables below show the 2019-2020 classroom teacher pay scale and the percentage increases for September 2020-2021.



Classroom Teachers

SPINE POINT	1 SEPT 2019 TO 31 AUG 2020
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Min M1	£24,373
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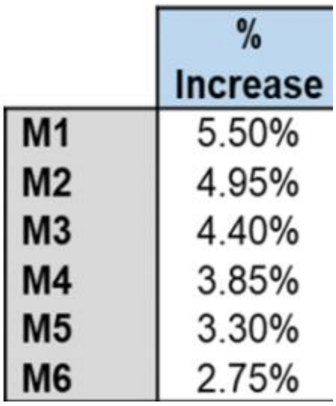
M5	£33,010
Max M6	£35,971



	% Increase
M1	5.50%
M2	4.95%
M3	4.40%
M4	3.85%
M5	3.30%
M6	2.75%

A recently appointed teacher will be on paid on spine point 2 from September 2020. What will her salary be?

method A:

100% + 4.95% = 104.95%

104.95 ÷ 100 = 1.0495

New salary = 1.0495 x £26,298

= £27599.751 = £27,600

method B:

Increase = 4.95% of £26,298

= 0.0495 x £26,298

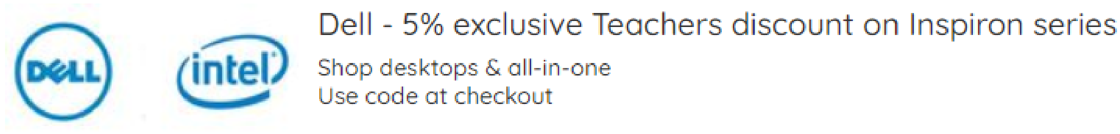
= £1301.751 = £1302

New salary = 26,298 + 1302

= £27,600

Now your turn 2.3b worked solution.

A teacher wishes to buy a new laptop and found this advert.



Dell - 5% exclusive Teachers discount on Inspiron series

Shop desktops & all-in-one

Use code at checkout

Source: https://www.discountsforteachers.co.uk/shopping/electrical/computing

What would the discounted cost be on a laptop costing £329?

method 1:

100% - 5% = 95%

95 ÷ 100 = 0.95

Discounted price = 0.95 x £329

= £312.55

method 2:

Discount = 5% of £329

= 0.05 x 329 = £16.45

Discounted price = £329 - £16.45

= £312.55

2.3 Further learning and support.

For method A https://corbettmaths.com/2012/08/21/multipliers-for-increasing-and-decreasing-by-a-percentage/
For method B https://corbettmaths.com/2012/08/21/increasing-or-decreasing-by-a-percentage/

2.4 Use percentages to compare data

Using percentages to compare data

Percentages are often useful when making comparisons between data.

Example

Total number of half days of unauthorised absences each term

	Term 1	Term 2	Term 3	Total number of absences
Year 9 (2017/18)	46	60	44	150
Year 9 (2018/19)	34	78	18	130

In which year was the percentage of term 2 unauthorised absences, out of total absences, the greater?

From the table, the term 2 absences for the year 2017/18 are 60 out of 150.

For the year 2018/19 the absences for term 2 were 78 out of a total absences of 130. Convert both of these fractions to percentages. The first can be done mentally:

$^{60}/_{150} = {}^{6}/_{15}$

$= {}^{2}/_{5}$

$= {}^{4}/_{10}$

$= 40\%$

For the second, convert the fraction first to a decimal, using a calculator if you need to, and then to a percentage.

$^{78}/_{130} = 78 \div 130$

$= 0.60$

$= 60\%$

The percentage of unauthorised absences in term 2 for 2018/19 = 60%. The percentage of unauthorised absences in term 2 for 2017/18 = 40%.

This means that the percentage of term 2 unauthorised absences out of total absences was greater for 2018/19.

Worked examples

Example 1

A pupil obtained the following marks in 3 tests.

	Test 1	Test 2	Test 3
Actual mark	95	39	61
Possible mark	150	60	100

In which test did the pupil achieve the best result?

To answer this question using percentages, first convert each fraction to a decimal, using a calculator where appropriate:

$^{95}/_{150}$ = 95 ÷ 150 = 0.63 to 2 decimal places
$^{39}/_{60}$ = 39 ÷ 60 = 0.65
$^{61}/_{100}$ = 61 ÷ 100 = 0.61

Now convert each decimal result above to a percentage in your head or multiply by 100 on the calculator.

0.63 = 63%

0.65 = 65%

0.61 = 61%

65% is the highest percentage. The pupil achieved the best result in test 2.

Selecting the best method of comparing data is a matter of judgement. However, with data presented as in the question above, percentages would normally be used.

Example 2

Four schools had the following proportion of pupils receiving free school meals.

School	Proportion of pupils receiving free school meals
A	⅑
B	0.12
C	11%
D	11 pupils out of 110

<u>Which school had the highest proportion of pupils receiving free school meals?</u>

Converting all the proportions to percentages enables a comparison to be made easily.

The proportion of pupils having free school meals in school A is given as the fraction ⅑.

1÷ 9 = 0.11 to two decimal places, or 11%

The proportion for school B is given as a decimal: 0.12 = 12%

The proportion for school C is given at 11%

The proportion for school D is given as 11 out of 110. This is the same as $^{11}/_{110}$. First convert this to a decimal:

11 ÷ 110 = 0.1 = 10%

School A	School B	School C	School D
11%	12%	11%	10%

So school B had the highest proportion of pupils receiving free school meals because 12% is the highest percentage.

Example 3

The numbers of year 13 pupils who left school to enter employment at the end of 2018 and at the end of 2019 are given in the table below.

Destination	2018	2019
Employment	50	60
Total number of Year 13 pupils	525	540
Percentage entering employment	9.5%	11.1%

By how many percentage points had the number of pupils entering employment increased between 2018 and 2019?

Finding the increase in percentage points is just a matter of comparing the percentages.

The percentage entering employment in 2018 was 9.5% and in 2019 was 11.1%.

The increase was 11.1% - 9.5% = 1.6%

The increase was 1.6 percentage points.

By what percentage did the numbers entering employment increase between 2018 and 2019?

To find the percentage increase, we need to think what any increase is to be compared to. There was an actual increase of pupils entering employment of 10. Compare this increase with the original number of pupils to get a measure of the significance of the increase.

To calculate the percentage increase that 10 more pupils represents, it is necessary to find 10 as a percentage of the number of pupils in employment last year, that is, 10 as a percentage of 50.

$^{10}/_{50} = 10 \div 50 = 0.2 = 20\%$

So, the number entering employment increased by 20% between 2018 and 2019.

Avoiding common errors

Most errors can be avoided by:

- memorising equivalent values for commonly used percentages, fractions and decimals;
- recognising the equivalence between decimals, fractions and percentages, for example, $^{35}/_{100}$, 0.35 and 35%;
- knowing how to convert from fraction into decimals and percentages into another form and back again;
- simplifying fractions to ease understanding and use; and
- clearly understanding the difference between a percentage increase or decrease and a percentage point increase or decrease

Now your turn 2.4

At a staff meeting, the head teacher presented the following table, showing the number of pupils in each class of year 7 who are having extra music lessons.

Class	**Number of pupils**	**Number of pupils having extra music lessons**
7x1	25	5
7x2	30	5
7x3	30	5
7x4	28	7
7y1	26	5
7y2	29	6
7y3	18	6
7y4	24	8

What half of year 7, x or y has the largest proportion of pupils having extra music lessons?

The answer is on the next page.

Now your turn 2.4 worked solution

At a staff meeting, the head teacher presented the following table, showing the number of pupils in each class of year 7 who are having extra music lessons.

Class	Number of pupils	Number of pupils having extra music lessons
7x1	25	5
7x2	30	5
7x3	30	5
7x4	28	7
7y1	26	5
7y2	29	6
7y3	18	6
7y4	24	8

What half of year 7, x or y has the largest proportion of pupils having extra music lessons?

x half: total in half: 25+30+30+28 = 113
total lessons : 5+5+5+7 = 22

converts to a decimal

$22 \div 113 \times 100 = 19.469\ldots$
$= 19\%$

converts decimal to percentage.

y half. total in half: 26+29+18+24 = 97
total lessons: 5+6+6+8 = 25

$25 \div 97 \times 100 = 25.773\ldots$
$= 26\%$

The y half has the largest proportion

2.4 Further learning and support.

Amounts as percentages https://corbettmaths.com/2012/08/21/expressing-one-quantity-as-a-percentage-of-another/

Percentage change https://www.drfrostmaths.com/videos.php?skid=54

2.5 Use ratio and proportional reasoning to solve problems

Conversions

Conversion involves changing information from 1 unit of measurement to another. For example, converting euros to pounds sterling (£), distances in kilometres to miles, or weights from pounds to kilograms. Most conversation problems can be solved using the unitary method. This is where a multiplier is used, with every 1 of the first unit is expressed as a multiple of the other. Often with exchange rates, the rate is expressed in unitary form.

Example 1

On a school trip to Spain the entrance fee to a museum for a group of pupils is 64 euros. If the exchange rate is 1.6 euros = £1, what is the group entrance fee in pounds sterling?

Divide the total entrance cost in euros by the numbers of euros equivalent to 1

pound:

64 ÷ 1.6

This calculation is made easier if you multiply both numbers by

10:

640 ÷ 16 = 40

So the group entrance fee in pounds sterling is £40.

Example 2

In this example the conversation is not in unitary form.

A teacher led a group of pupils on a camping trip to France. The teacher wanted to arrange a cycle ride of approximately 8 miles for the pupils to a local village and back. The signpost at their campsite showed the distance to the following villages:

La Pet 7km
Fouin 6km
Travin 5km

Taking 5 miles to be equivalent to 8 kilometres, which of the three villages should the teacher choose as the destination for the cycle ride?

The return distance from the campsite to La Pet is 2 x 7km =14km

Convert this to miles by dividing by 8, converting it to its unitary

form, and multiplying by 5:

14 ÷ 8 x 5 = 8.75 miles

The return distance from the campsite to Fouin is 2 x 6 =12km

Convert this to miles by dividing by 8 and multiplying by 5:

12 ÷ 8 x 5 = 7.5

The return distance from the campsite to Travin is 2 x 5km = 10km

Convert this to miles by dividing by 8 and multiplying by

5: 10 ÷ 8 x 5 = 6.25 miles

You can also consider that the distance to Travin is less than the distance to Fouin in kilometres. Given that the return distance from the camp to Fouin is 7.5 miles, it is clear that Travin is not the right answer.

You can see that 7.5 miles is closer to 8 miles than 8.75 miles, so Fouin is the village to choose as the destination for the cycle ride.

Example 3

For some conversions a formula may be used. See also 2.7……or using simple formula.

An approximate conversion from kilometres to miles is given by the

formula

$M = \frac{5}{8}K$

where K represents the number of kilometres and M represents the number of miles.
To convert 12 kilometres into miles using the formula, you can substitute as follows:

$M = \frac{5}{8}$ of 12

$M = \frac{60}{8}$ (multiplying 5 by 12)

$M = 7.5$ (by dividing by 8)

So 12 kilometres is approximately 7.5 miles.

To convert 60 miles into kilometres substitute into the formula as

follows:

$60 = \frac{5}{8} K$

$60 \times 8 = 5 K$ (multiply both sides by 8)

$480 = 5 K$

$K = 480 \div 5$ (divide both sides by5)

$K = 96$

So 60 miles is approximately 96 kilometres

Proportion

A proportion is used to describe the relationship of some part of a whole to the whole itself and is usually given as a fraction.

Example
In a class there are 20 girls and 10 boys. The ratio of girls to boys is 20:10, or 2:1, and the proportion of girls in the class is 20 out of 30 or 20⁄30 = 2⁄3.

See also 2.6 simplifying ratio

Worked examples

Example 1

An infant school has 180 pupils, of whom 35 go home for lunch. What proportion of pupils has lunch at school?

35 out of 180 go home.

So 180 - 35 = 145 stay at school for lunch.

As a proportion this is represented as 145 out of 180

145⁄180 = 29⁄36 (dividing top and bottom by 5)

So the proportion that stays at school for lunch is 29⁄36.

Example 2

Three schools have the same proportion of pupils receiving free school meals, $\frac{8}{75}$. If the numbers of pupils receiving free school meals in each of the 3 schools are 16, 40 and 56 respectively, how many pupils in each school do not receive free school meals?

The proportion of pupils receiving free school meals is 8 out of every 75, or $\frac{8}{75}$.

To work out the number of pupils in each of the 3 schools who do not receive free school meals, you need to find out the total number of pupils in each school. You can do this using the proportion $\frac{8}{75}$ by relating the 16, 40, 56 to the 8 out of every 75 proportion given.

8 out of 75 is 16 out of 150 (the 8 has been doubled. To maintain the same proportion the 75 must also be doubled, giving 150)

is 40 out of 375 (the 8 has been multiplied by 5; 75 x 5 = 375)

is 56 out of 525 (the 8 has been multiplied by 7; 75 x 7 = 525)

So the total number of pupils in each school is 150, 375 and 525 respectively.

To find the numbers of pupils who do not have free school meals in each school, simply subtract the number of pupils getting free school meals from the total number of pupils.

School 1

150 pupils of which 16 receive free school meals,
so 150 - 16 = 134 pupils do not receive free school meals.

School 2

375 pupils of which 40 receive free school meals,
so 375 - 40 = 335 pupils do not receive free school meals.

School 3

525 pupils of which 56 receive free school meals,
so 525 - 56 = 469 pupils do not receive free school meals.

Avoiding common errors

Most common errors can be avoided by remembering that proportion can be scaled by multiplying or dividing both parts by the same number. Also remember, ratio compares parts with parts, whereas proportion compares part out of the whole.

Now your turn 2.5a

A teacher plans a school trip to Brussels, which involves using a ferry from Ostend. The teacher wants to be in Ostend no later than 18:00. She expects their coach to travel from Brussels to Ostend, a distance of 120km, at an average of 50 miles per hour.

Using the approximation of 5 miles equals 8 kilometres, what is the latest time that the coach should leave Brussels? Give your answer using the 24-hour clock.

Now your turn 2.5b

A sixth form tutor organises a sponsored swim.
Five students in the tutor group take part in the swim and
they decide to donate the money to two charities, X and Y, in the ratio 2 : 1.

The tutor records the number of lengths they swim and the amount of money they raise.

Student	**Number of lengths**	**Amount of money raised (£)**
A	10	9.50
B	9	11.20
C	10	13.20
D	8	10.05
E	9	12.15

How much money is given to charity X?

The answers are on the next page.

Now your turn 2.5a worked solution.

A teacher plans a school trip to Brussels, which involves using a ferry from Ostend. The teacher wants to be in Ostend no later than 18:00. She expects their coach to travel from Brussels to Ostend, a distance of 120km, at an average of 50 miles per hour.

Using the approximation of 5 miles equals 8 kilometres, what is the latest time that the coach should leave Brussels? Give your answer using the 24-hour clock.

If 5 miles = 8 km, then 0.625 miles = 1 km (÷8 to get 1km)

120 km × 0.625 = 75 miles (OR 120 ÷ 8 × 5 = 75)

It will take $1\frac{1}{2}$ hours to travel 75 miles at 50 mph

50 + 25 = 75

1h + ½h = $1\frac{1}{2}$h.

$1\frac{1}{2}$ hours before 18:00 is 16:30

Now your turn 2.5b worked solution.

A sixth form tutor organises a sponsored swim.
Five students in the tutor group take part in the swim and
they decide to donate the money to two charities, X and Y, in the ratio 2 : 1.

The tutor records the number of lengths they swim and the amount of money they raise.

Student	**Number of lengths**	**Amount of money raised (£)**
A	10	9.50
B	9	11.20
C	10	13.20
D	8	10.05
E	9	12.15

How much money is given to charity X?

Total money raised: 9.50 + 11.20 + 13.20 + 10.05 + 12.15
= £56.10

If the money is shared in the ratio 2:1, then charity X receives a proportional share of $\frac{2}{3}$

2+1 = 3 shares in total
2 for X and 1 for Y.

56.10 ÷ 3 x2 = £37.40

2.5 Further learning and support.

http://mrbartonmaths.com/topics/ratio-and-proportion/best-value-recipes-exchange-rates/videos.html

2.6 Simplify ratios

Ratio

A ratio is a comparison between 2 quantities.

Example 1

If there are 20 girls and 10 boys in a class, the ratio of girls to boys is 20 to 10. This is written as 20:10. Ratios, like fractions, can be simplified, so 20:10 could be written as 2:1. In this example, the 2 'quantities' being compared are girls and boys. 20 girls are compared with 10 boys. This comparison is summarised by using the colon symbol (:).

The quantities are always compared in the order of the statement. In the example, girls are compared with boys. So, the ratio of girls to boys is 20:10. If the boys are compared with the girls, the ratio is 10:20, or 1:2.

The ratio 20:10 may be simplified to 2:1. The ratio 2:1 expresses the fact that there are 2 girls for every 1 boy in the class. The original ratio 20:10 not only expresses the fact that there are 20 girls for every 10 boys in the class, but also the fact that the class contains 30 pupils (see the page on proportion).

In the example above, like quantities are being compared as both girls and boys are pupils. Ratios must always compare 2 values of the same type of data. So if 6 hours are to be compared with 1 day, the ratio is 6:24 because there are 24 hours in a day.

Example 2

A school has 360 key stage 3 (KS3) pupils and 200 key stage 4 (KS4) pupils. Show this information as a ratio.

The ratio which compares the number of KS3 pupils with the number of KS4 pupils

is: 360:200

This can be simplified by dividing by 10

to: 36:20

This ratio shows that for every 36 KS3 pupils there are 20 KS4

pupils. Simplifying this further by dividing each number by 4 gives:

9:5

The ratio 9:5 states that for every 9 KS3 pupils there are 5 KS4

pupils. The ratio of KS3 pupils to KS4 pupils is 9:5.

Example 3

In a 25 hour week of lessons a pupil has 5 hours of science and 2.5 hours of design and technology (D&T). Find the ratio of D&T hours to science hours and the ratio of science hours to the total lesson hours. Express both as ratios in their lowest form.

The ratio of D&T hours to science hours is:

2.5 to 5 hours or 2.5:5, which is 1:2 in its simplest form. So for every 2.5 hours of D&T, there were 5 hours of science.

The ratio of science hours to total lesson hours is: 5 to 25 or 5:25, which is 1:5 in its lowest form.

Avoiding common errors

Most common errors can be avoided by simplifying the ratio and ensuring the numbers in the ratio are in the same order as the required comparison.

Now your turn 2.6a

There are 35 schools in a local authority.

Twenty-eight schools have been inspected in the

last 4 years.

What is the ratio of inspected to not inspected? Give your answer in its simplest form.

Now your turn 2.6b

A teacher took a group of students to an aquarium whilst visiting France. The group of 51, consisted of 48 students and three staff including the teacher. The aquarium offers free staff entry based on the number of students. The ratio of students to staff for free entry is 15:1 Will all the staff gain free admission?

The answers are on the next page

Now your turn 2.6a worked solution.

There are 35 schools in a local authority.

Twenty-eight schools have been inspected in the

last 4 years.

What is the ratio of inspected to not inspected? Give your answer in its simplest form.

Schools not inspected. 35-28=7

Inspected : not inspected

28 : 7

÷7 ÷7

4 : 1

dividing both numbers by the 7 as its the largest number that goes into 28 and 7

4:1 means that for every 4 inspected schools, one is yet to be inspected.

Now your turn 2.6b worked solution.

A teacher took a group of students to an aquarium whilst visiting France. The group of 51, consisted of 48 students and three staff including the teacher. The aquarium offers free staff entry based on the number of students. The ratio of students to staff for free entry is 15:1

Will all the staff gain free admission?

method 1

51-3= 48

student : staff

48 : 3

÷3 ÷3

16 : 1

The Aquarium offers 15:1 which is lower, so all staff will go in free.

method 2

Student : staff

15 : 1

x3 x3

45 : 3

The Aquarium will let in 3 staff free for 45 students. The group has 48 paying students so all 3 staff can gain free entry

2.6 Further learning and support.

See unit 7 http://www.cimt.org.uk/projects/mepres/book8/book8int.htm

And https://www.mathsgenie.co.uk/writing-simplifying-ratio.html

2.7 Solve mathematical problems by breaking them down into a series of simpler steps and selecting appropriate operations (+,-,÷,x) or using simple formulae.

Formulae

A formula is a statement in words or symbols showing the relationship between 2 or more variables, eg, degrees Celsius and degrees Fahrenheit. Formulae are useful general statements which can be applied to different numbers on different occasions.

Example

You could write or use a formula to represent the total cost of a number of packs of magazines at £2 per pack and additional individual copies at 40p each. You would need to have a variable P for the number of packs of magazines, a variable N for the number of individual copies required, and a variable C for the cost in pounds.

So for any order of packs and individual copies, the total cost is found

by: (P x £2) + (N x 40p)

The cost of individual copies is in pence, but the cost of the packs and the total cost are both in pounds. When constructing a formula you need to make sure that all the units are the same. To keep them all in pounds, write 40p as £0.4. Then the formula is:

$C = (P \times 2) + (N \times 0.4)$

or

$C = 2P + 0.4N$

To find the total cost for an order of 7 packs and 3 individual copies, replace P and N with the appropriate numbers. In this case P is 7 and N is 3.

$C = (7 \times 2) + (3 \times 0.4)$

$= 14 + 1.2$

$= 15.2$

So the total cost is £15.20.

Many GCSE syllabuses are examined by testing several components which are said to be ‘weighted’. For example, in GCSE science (1999-2000 syllabuses) the practical tasks are marked out of 30 and the score, when doubled, counts towards the final mark. So here, the practical tasks have double weighting. Then a score out of 3 for spelling, punctuation and grammar (SPaG) is included. A formula is applied to the pupils’ marks to produce a final mark out of 63.

There are three variables: the practical mark, the SPaG mark, and the total

mark. Call the practical mark P, the SPaG mark S, and the total mark T.

The formula is:

$T = (P \times 2) + S$ or $T = 2P + S$

1 pupil got 24 for the practical mark and 2 for the SPaG. So,

$P = 24, S = 2$

and $T = (2 \times 24) + 2$

$= 50$

So this pupil got a mark of 50 out of a possible 63.

Worked examples

Example 1

An examination consisted of 2 papers. Paper 1 is marked out of 80 with a SPaG mark out of 4 added. Paper 2 is marked out of 120, with a further SPaG mark out of 4 added. The final mark is obtained by adding together the 4 marks and giving this total as a percentage, rounded to the nearest whole number.

In Paper 1, a pupil gets 57 out of 80 and for SPaG 3 out of 4. In Paper 2, he gets 78 out of 120 and for SPaG 2 out of 4. What will his final mark be, expressed as a percentage?

There are 2 stages required to answer this question.

Stage 1

Paper 1 is marked out of 80 and the SPaG mark out of 4, so the maximum possible mark is 84.

Paper 2 is marked out of 120 and SPaG mark out of 4, so the maximum possible mark is 124.

The total maximum possible maximum mark from both papers is

84 + 124 = 208.

The formula required to obtain candidates' final marks out of 208 is:

Final mark = (mark + SPaG for paper 1) + (mark + SPaG for paper 2)

Inserting the pupil's marks into the formula gives:

Final mark = (57 + 3) + (78+2)

= 60 + 80
= 140

Therefore the pupil achieved 140 out of 208 or 140/208.

Stage 2

Convert 140/208 to a percentage.

140 ÷ 208 = 0.67307692 = 67.31%

= 67% to the nearest whole number

Therefore the pupil's final mark is 67%.

Example 2

A teacher sets her pupils 3 tests in year 7. She weights the marks in the second test by 3 and those of the third test by 5. Each test is marked out of 100, so the total possible marks are 100 + 300 + 500 = 900.

Call the marks gained in the first test T1, those in the second test T2, and in the third test T3.

The total mark T is found by the formula:

$$T = T1 + (T2 \times 3) + (T3 \times 5)$$

or

$$T = T1 + 3T2 + 5T3$$

A pupil gets 15 for the first test, 21 for the second test, and 58 for the third test.

$T1 = 15$, $T2 = 21$ and $T3 = 58$

$$T = 15 + (21 \times 3) + (58 \times 5)$$

$$= 15 + 63 + 290$$

$$= 368$$

So his total mark is 368 out of 900.

To turn this into a percentage:

$$368 \div 900 = 0.4088$$

= 40.88 or 41% to the nearest whole number

Avoiding common errors

Most common errors can be avoided by:

- ensuring calculations are carried out in the correct order (carry out multiplication and division before addition and subtraction);
- knowing that a symbol (usually a letter) is simply a shorthand way of writing a longer statement, eg (A) represents the marks out of 65 on test 1, (B) represents the number of books ordered; and
- ensuring that you know what the units are before you calculate, so you don’t confuse pounds with pence, metres with centimetres, and so on.

Weighting

Example 1

In a GCSE examination which has 3 components (A, B and C), the raw marks for component A are weighted twice as much as the raw marks for component B, and those for component C are weighted half as much as the raw marks for component B. Thus, marks for A would be doubled and those for C halved before totalling, giving the formula:

Final mark = (2 x A) + B + ½ C

A pupil achieved the following marks in tests A, B and C.

Test	A	B	C
Raw mark	68	28	5

The pupil's weighted score was calculated using the following formula:

Weighted score =	(A x 60) 100	+	(B x 30) 80	+C

What was the pupil's weighted score? Give your answer to the nearest whole number.

Weighted score =	(68 x 60) 100	+	(28 x 30) 80	+5
	= 40.8 + 10.5 + 5 = 56.3			

Example 2

A GCSE examination consists of 2 papers. Paper 1 has a maximum raw mark of 40 and a final weighting of 30%. Paper 2 has a maximum raw mark of 160 and a final weighting of 70%, as shown in the table:

	Maximum raw mark	**Weighting %**
Paper 1	40	30
Paper 2	160	70

Find the final weighted mark for a pupil who gained 27 marks in paper 1 and 93 marks on paper 2.

For paper 1: the score of 27 out of 40 is equivalent to a weighted percentage of

$^{27}/_{40}$ x 30% = 20.25%, rounded to 20.3%

For paper 2: the score of 93 out of 160 is equivalent to a weighted percentage of

$^{93}/_{160}$ x 70% = 40.68%, rounded to 40.7%

So the final weighted mark is 20.3% + 40.7% = 61%

More problem-solving examples.

Example 1

A teacher is planning a group outing to see a play in a nearby city. She has been given details of costs and travel.

There are 25 in the group, including pupils and teachers. A group booking for 25 theatre tickets costs £185.

Return train tickets cost £5.65 each.

<u>How much will each person have to pay for the outing to cover the cost of travel and theatre ticket?</u>

The number of people in the group is 25. The total cost of the theatre tickets is 185

The cost of each theatre ticket is:

Total cost of tickets ÷ number of people in the group = individual ticket price

185 ÷ 25 = £7.40

The cost of each return train ticket is £5.65.

The amount each person will have to pay for the outing is:

7.40 + 5.65 = £13.05.

Example 2

A newly qualified teacher attends a series of training courses during the induction year. Each course has three separate sessions on different dates.
The table shows the total travelling distance for each course.
The teacher can claim travelling expenses of
32.5 pence per mile for the first 100 miles in each school year and 27.5 pence per mile above 100 miles, in addition to any train fares.

The teacher travels by train on the 17/12/19 and
the 11/04/20 at a cost of £11.50 and £4.70 respectively.

	Date	**Total travelling distance (miles)**
Course A	05/10/19	27
	16/10/19	27
	17/12/19	Train
Course B	18/01/20	32
	08/02/20	32
	28/03/20	32
Course C	11/04/20	Train
	02/05/20	18
	09/05/20	18

<u>How much does the teacher claim for the travelling expenses for attending these courses?</u>

Total mileage: (2 × 27) + (3 × 32) + (2 × 18) = 54 + 96 + 36 = 186 miles.

Total mileage claim: (100 × 32.5p) + (86 × 27.5p) = £56.15.
Total rail fares: £11.50 + £4.70 = £16.20. Total fares = £56.15 + £16.20 = £72.35.
Notes:
The number of miles that should be claimed at the lower rate is the total mileage (186) less the basic mileage (100).
The overall claim is the total of the mileage claim and the rail fares.

Example 3

A school parents' evening starts at 16:30 on each of two consecutive days.

A teacher has a total of 24 appointments lasting 10 minutes each and takes a 20 minute break each evening. The teacher filled all the available appointment slots on the first evening and finished at 19:00.

<u>What is the earliest time the teacher can expect to finish on the second evening?</u>
<u>Give your answer using the 24-hour clock.</u>

The total time on the first day is: 16:30 to 19:00 = 2 hours 30 minutes.
This time includes a break time of 20 minutes.
The time available for appointments is:
2 hours 30 minutes – 20 minutes = 2 hours 10 minutes.
This time expressed in minutes is 130 minutes.
Each appointment takes 10 minutes.
The number of appointments on the first day was: 130 ÷ 10 = 13.

The total number of appointments is 24.
The number of appointments on the second day is: 24 – 13 = 11.
The appointments on the second day will end (2 × 10).
= 20 minutes before the end time on the first day.
This time is: 19:00 (24-hour clock) – 20 minutes = 18:40 (24-hour clock).

Further help

A common mistake is to forget there is a break of 20 minutes on each day.

Another common mistake is to use the 24-hour clock as a decimal number, i.e. 100 minutes in the hour.

Now your turn 2.7a

In order to predict pupils' achievement in a GCSE subject, a teacher produced the following table.

The table shows the marks achieved by 3 pupils in coursework and in a practice examination. Using the previous year's results, the teacher set a minimum final percentage mark of 55 for a predicted grade 5.

Final percentage mark =

$$\frac{\textbf{coursework percentage mark} + (\textbf{3} \times \textbf{practice examination percentage mark})}{\textbf{4}}$$

Pupil	Coursework mark out of 60	Practice examination mark out of 100
X	22	45
Y	21	60
Z	30	58

Which pupil is predicted to achieve a grade 5?

Now your turn 2.7b

A teacher researches the cost of 15 packs of wooden shapes to use for a problem solving activity

Source	Cost per pack (£)	Offers	Postage (£)	Total (£)
Catalogue	3.40	5 packs for the price of 4	No postage charge	
Internet	2.50	None	1.60 per 5 packs	

Complete the table.

Now your turn 2.7c

An ICT teacher compares the cost of building a paper-based ICT portfolio with the cost of using commercial e-portfolio software. The number of pupils on the course is 125.

On average, each paper-based portfolio includes 75 printed pages.

Costs are:	printing - 2.5p per page
	ring binder - 75p.

The total cost of the e-portfolio software is £250.00 per year.

How much money would the school save by using the e-portfolio software? Give your answer to the nearest pound.

Now your turn 2.7d

A parents' evening is planned to last from 16:15 to 19:00.
Within that time teachers will have a break from 17:30 until

17:45. Each appointment is scheduled to last 8 minutes.

What is the maximum number of appointments each teacher can have?

The answers are on the next page.

Now your turn 2.7a worked solution.

In order to predict pupils' achievement in a GCSE subject, a teacher produced the following table.

The table shows the marks achieved by 3 pupils in coursework and in a practice examination. Using the previous year's results, the teacher set a minimum final percentage mark of 55 for a predicted grade 5.

Final percentage mark =

$$\frac{\textbf{coursework percentage mark + (3 × practice examination percentage mark)}}{\textbf{4}}$$

Pupil	Coursework mark out of 60	Practice examination mark out of 100
X	22	45
Y	21	60
Z	30	58

Which pupil is predicted to achieve a grade 5?

Pupil X.

coursework % = $22 \div 60 \times 100 = 36.7\%$ (1 dp)

Final % = $(36.7 + 3 \times 45) \div 4 = 42.9\%$ (1 dp)

not a grade 5.

Pupil Y

coursework % = $21 \div 60 \times 100 = 35\%$

Final % = $(35 + 3 \times 60) \div 4 = 53.8\%$

not a grade 5.

Should be pupil 2, as other two not enough, but to check:

Pupil 2

coursework % = 30 ÷ 60 × 100 = 50%

Find % = (50 + 3× 58) ÷ 4 = 56%.

Pupil 2 is predicted to get a grade 5.

Now your turn 2.7b worked solution.

A teacher researches the cost of 15 packs of wooden shapes to use for a problem solving activity.

Source	**Cost per pack (£)**	**Offers**	**Postage (£)**	**Total (£)**
Catalogue	3.40	5 packs for the price of 4	No postage charge	40.80
Internet	2.50	None	1.60 per 5 packs	42.30

Complete the table.

Catalogue. 15 ÷ 5 = 3, so get 3 free packs.

3.40 × (15 − 3) = 3.40 × 12 = £40.80

Internet 2.50 × 15 = £37.50

15 ÷ 5 = 3

postage = 3 × 1.60 = £4.80

total = 37.50 + 4.80 = £42.30

Now your turn 2.7c worked solution.

An ICT teacher compares the cost of building a paper-based ICT portfolio with the cost of using commercial e-portfolio software. The number of pupils on the course is 125.

On average, each paper-based portfolio includes 75 printed pages.

Costs are: printing - 2.5p per page
ring binder - 75p.

The total cost of the e-portfolio software is £250.00 per year.

How much money would the school save by using the e-portfolio software? Give your answer to the nearest pound.

Printing per pupil = $75 \times 0.025 = £1.875$ ÷100 to change pence to £

cost per pupil = $1.875 + 0.75 = £2.625$

total cost = $125 \times 2.625 = 328.125$

$= £328$ round only at final stage

Saving: $£328 - 250 = £78$

Now your turn 2.7d worked solution.

A parents' evening is planned to last from 16:15 to 19:00.

Within that time teachers will have a break from 17:30 until

17:45. Each appointment is scheduled to last 8 minutes.

What is the maximum number of appointments each teacher can have?

To break:

16.15 to 17.30 is 45+30 = 75 mins.

75 ÷ 8 = 9.375 = 9 appointments.

After break.

17.45 to 19.00 is 15 + 60 = 75 mins

= 9 more appointments.

total appointments = 9+9 = 18

2.7 Further learning and support.

Substitution into formula https://www.youtube.com/watch?v=9rUI8ro9jho

Calculation problems https://www.mathsgenie.co.uk/calculationproblems.html

Weightings (advanced),see page 62 (P58 of booklet)
https://www.cimt.org.uk/projects/mepres/allgcse/pbtxt.pdf

For time related problems https://corbettmaths.com/2014/10/12/converting-hours-to-hoursminutes/

3.1 Summarise and compare data sets by finding the mean, mode, median, interquartile range and range

Averages

An average is a single number which is used to represent a group of values collected for a particular purpose.

Mean, median and mode

Mean

The mean is the most commonly used 'average' and the 1 often referred to in everyday conversation, eg the average wage. The mean is usually used when the data involved is fairly evenly spread, and there are no exceptional cases that are much higher or lower than the rest. If a few exceptional results exist, the mean may give a misleading impression, because it takes account of all the data given. It is found by adding together all the data values in a set of data and then dividing this total by the number of values in the set.

Example

In a test, a group of 11 pupils scored the following

marks:

5, 10, 3, 4, 4, 8, 4, 3, 11, 9, 5

The mean mark is found by adding together all the marks and dividing the total by the number of pupils.

(5+10+3+4+4+8+4+3+11+9+5)÷11

= 66 ÷ 11

= 6

Therefore the mean mark is 6.

Median

The median is the middle value of a set of data when placed in order. It can be found with little or no calculation. The median is particularly useful when the data has a wide spread, as the middle value is not affected by exceptional cases. To find the median age of a group of 11 pupils it is only necessary to arrange their ages in order, and the age of the middle child (the sixth child) is the median. This means that for the median there are as many values in the dataset above the median as there are below it. The median for any data can also be found by drawing a cumulative frequency graph. See the page on cumulative frequency for more information.

Example

The median mark is the middle value in the group of marks when arranged in order of size.
In order of size the marks are:

3	3	4	4	4	5	5	8	9	10	11
					⇧					
Middle value (Median)										

There are 11 numbers in this set of data. The sixth number is the middle value or median. In this example the median is 5.

Note: When there is an even number of values, the median is found half way between the 2 middle values.
For example, if the test results for a particular pupil are: 12, 13, 16, 17, 21 and 25, then 16 and 17 are the middle values. The median is the mean of the 2 values, or half way between the 2 values. That is (16 + 17)/2. The median in this example is 16.5.

Mode

The mode is the most frequently occurring value in a set of data. For example shoe manufacturers are not interested in the mean or median value of shoe sizes but they may want to know the size most frequently sold.

Example

The mode is the value which occurs most often in the set of marks. For the set of test marks: 5, 10, 3, 4, 4, 8, 4, 3, 11, 9, 5
The score which occurs most often is 4 with 3 pupils scoring 4 marks, so the modal mark, or mode, is 4.

Worked examples

Example 1

The table below records the amount of time, rounded to the nearest quarter of an hour, each member of a class spends on homework in hours.

Time spent on homework during the week ending 24/1/2019

Pupil	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Hours	2¾	2½	1½	1½	3	1¾	1½	¾	1¾	2¾	2½	2	1½	2½	1¾

<u>What is the mean amount of time pupils spend on homework?</u>

To find the mean, first add all the values together. The answer is 30.

Then divide by the number of items, in this case, 15.

So the mean is 30/15 = 2.

Therefore the mean amount of time spent on homework during the week is 2 hours.

The teacher may also be interested in the most common amount of time spent by pupils, the mode:

Time in hours	¾	1½	1	2	2½	2¾	3
Number of pupils=15	1	4	3	1	3	2	1

The amount of time pupils spent varies a good deal. However, 4 pupils spent 1.5 hours.

Therefore the mode is 1.5 hours.

Example 2

A teacher sets a test for 20 pupils that is marked out of a possible total of 20 marks. The teacher wants to know the median mark for the test.

Ordering the test scores gives:

19	19	19	18	18	17	16	15	15	15	14	14	14	12	12	11	10	10	2	1
									↑	↑									

As there are 20 marks there will be 2 middle marks, the 10th and 11th. These are: 15 marks and 14 marks.

The median is half way between them, or the mean of the 2 marks.

So the median is 14.5 marks.

Example 3

The set of data below is the GCSE business studies grade results for a group of 70 pupils.

Business studies GCSE results

7	4	4	U	5	6	3
1	8	4	6	2	7	1
2	5	5	3	5	5	4
4	2	2	5	5	4	U
4	4	7	U	8	5	4
3	4	1	4	2	5	4
6	3	6	U	1	6	5
5	3	2	5	5	2	2
5	2	3	8	6	6	1
4	3	3	1	6	5	1

The teacher wants to know what the average performance is and chooses the mode as the most appropriate average, that is, the grade that the most pupils achieve.

To find the mode the data needs to be sorted. The number of times each grade occurs is counted and entered as below:

Grade	8	7	6	5	4	3	2	1	U
Number of pupils achieving grade	3	3	8	15	13	8	9	7	4

The grade that occurs most frequently is 5. So the modal grade is 5.

The teacher might also be interested in the average (mean) points score. To calculate the mean the U's will counted as grade "0" points and the other grades will have the same points as grade.

The mean can be calculated by:

- multiplying the points score by the number of pupils at each grade
- finding the total
- dividing by the number of pupils (70):

(3x8 + 3x7 + 8x6 + 15x5 + 13x4 + 8x3 + 9x2 + 7x1 + 4x0) ÷ 70

= 269 ÷ 70

= 3.8 to 1 d.p.

The average (mean) points score is 3.8.

The nearest grade to 3.8 points is a grade 4.

This example shows that the mean and the mode do not necessarily give the same result. Both can be useful in this case, but are giving the teacher different information.

One tells the teacher the most common grade, while the other gives an average (mean) points score which might be useful for comparison with other subjects. The fact that the mean is less than the mode indicates that the results as a whole are likely to be spread out among the lower grades as shown in the second table above.

Avoiding common errors

Whenever the word 'average' is used, make sure you understand which average is being referred to in the context of the set of data: is it the mean, median or mode?

When finding the median, make sure that the data is arranged in ascending or descending order.

Range

Range is a measure of the spread of data. It is the difference between the largest and the smallest values. It is used, eg, in looking at the results of a test, to see what range of the available marks has been achieved. To find the range of a set of data, take the smallest value from the largest value.

Worked examples

Example 1

In a set of tests, 2 pupils scored the following marks out of 10:

John: 5, 6, 4, 5, 5, 6, 5, 7

Sally: 3, 2, 3, 6, 5, 8, 6, 9

The lowest mark John scored is 4 and the highest mark he scored is 7, so the range of John's marks is 3 (from 4 to 7).

Sally's marks are between 2 and 9 making the range for Sally's marks 7.

From this we can see that Sally's marks are more widely spread than John's marks. John's marks are fairly consistent, whereas Sally obtains some high and some low marks.

Example 2

The marks scored by a group of pupils in an end of term test marked out of 80 are as follows.

37, 45, 53, 61, 70, 50, 48, 29, 52, 59

The teacher was interested in the spread of marks. What is the range of the set of marks?

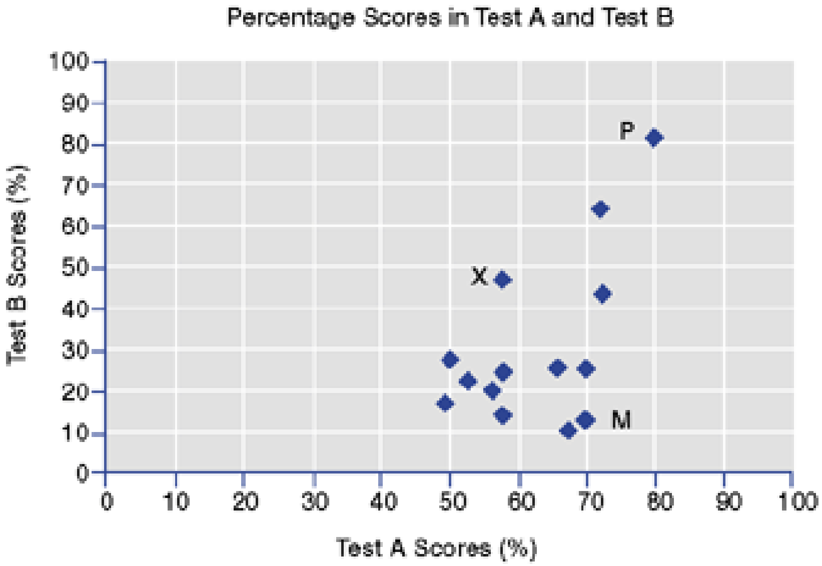
The lowest mark is 29

The highest mark is 70

The difference: 70 - 29 = 41

The range of marks is 41. This result shows wide variation in the group's performance on this test.

Example 3



The scatter graph shows the percentage scores of a group of pupils in test A and test B. Each dot on the graph represents 1 pupil. So pupil X scored 58 in test A and 48 in test B. Pupils' scores might be plotted on a scatter graph to see whether there is a correlation between the marks on the 2 tests. It shows that pupil P scored well in both tests, but that pupil M scored well on test A but badly on test B. The range helps in the analyses of tests to see to what extent they distinguish between the high achieving and low achieving pupils.

<u>Which test (A or B) had the greater range?</u>

To find the range of marks in test A, look along the x axis (horizontal) and read off the first and last values.

The lowest percentage score for test A is 49. The highest percentage score for test A is 80. The range of percentage scores for test A is 80 - 49 = 31.

To find the range of scores in for test B, look up the y axis (vertical) and read off the first and last values. The lowest percentage score for test B is 10. The highest percentage score for test B is 81. The range of percentage scores for test B is 81 - 10 = 71.

Test A percentage range is 31.Test B percentage range is 71.

The greater range is test B.

So in test B, a wider range of percentage scores were obtained.

Interquartile Range

See the box and whisker diagram notes within section 3.4

Now your turn 3.1a

As part of a review of performance, a geography teacher prepared a table of marks for eight pupils from a series of tests throughout the year.

Test marks in geography (marks out of 80)						
Pupil	**Test 1**	**Test 2**	**Test 3**	**Test 4**	**Test 5**	**Mean for pupil**
A	45	42	35	21	45	37.6
B	38	40	48	35	52	42.6
C	41	51	44	56	67	
D	48	58	62	70	58	59.2
E	25	28	34	35	42	32.8
F	15	21	28	19	27	22.0
G	40	29	35	38	41	36.6
H	52	59	68	35	70	56.8
Range	**37**	**38**	**40**	**51**		

Complete the table.

Now your turn 3.1b

A teacher produced the following table to show the marks achieved in an end of year geography test by pupils in three Year 7 classes.

Marks (Percentage)			
	Range	**Median**	**Mode**
Class A	60	50	72
Class B	28	50	68
Class C	85	60	70

Tick all the true statements:

☐ Some pupils in Class A achieved less than 12%.

☐ At least one pupil in Class C achieved less than 20%.

☐ All pupils in Class B achieved at least 40%.

The answers are on the next page.

Now your turn 3.1a worked solution.

As part of a review of performance, a geography teacher prepared a table of marks for eight pupils from a series of tests throughout the year.

Test marks in geography (marks out of 80)						
Pupil	**Test 1**	**Test 2**	**Test 3**	**Test 4**	**Test 5**	**Mean for pupil**
A	45	42	35	21	45	37.6
B	38	40	48	35	52	42.6
C	41	51	44	56	67	
D	48	58	62	70	58	59.2
E	25	28	34	35	42	32.8
F	15	21	28	19	27	22.0
G	40	29	35	38	41	36.6
H	52	59	68	35	70	56.8
Range	**37**	**38**	**40**	**51**		

Complete the table.

Mean for pupil C: $(41+51+44+56+67) \div 5 = 259 \div 5 = 51{\cdot}8$

Range for test 5: $70 - 27 = 43$

Now your turn 3.1b worked solution.

A teacher produced the following table to show the marks achieved in an end of year geography test by pupils in three Year 7 classes.

Marks (Percentage)			
	Range	**Median**	**Mode**
Class A	60	50	72
Class B	28	50	68
Class C	85	60	70

Tick all the true statements:

☐ Some pupils in Class A achieved less than 12%.

☑ At least one pupil in Class C achieved less than 20%.

☑ All pupils in Class B achieved at least 40%.

class A: The mode is 72, so at least one student scored 72%. Using the range 72-60=12%. The lowest possible score is 12%, assuming the mode is also the highest: False.

class C: The highest possible mark is 100%. Using the range 100-85 = 15%, the highest possible lowest value 15% is less than 20%: True

class B: The mode is 68, so at least one student scored 68%. Using the range, 68-28= 40%. So the lowest mark cannot be lower than 40%: True

3.1 Further learning and support.

http://www.cimt.org.uk/projects/mepres/book9/bk9i8/bk9_8i2.html

https://www.mathsgenie.co.uk/averages.html

For finding the quartiles https://www.drfrostmaths.com/videos.php?skid=383

3.2 Interpret and draw conclusions from data presented in tables, such as a two-way table, a bar chart or as a pie chart

Bar charts

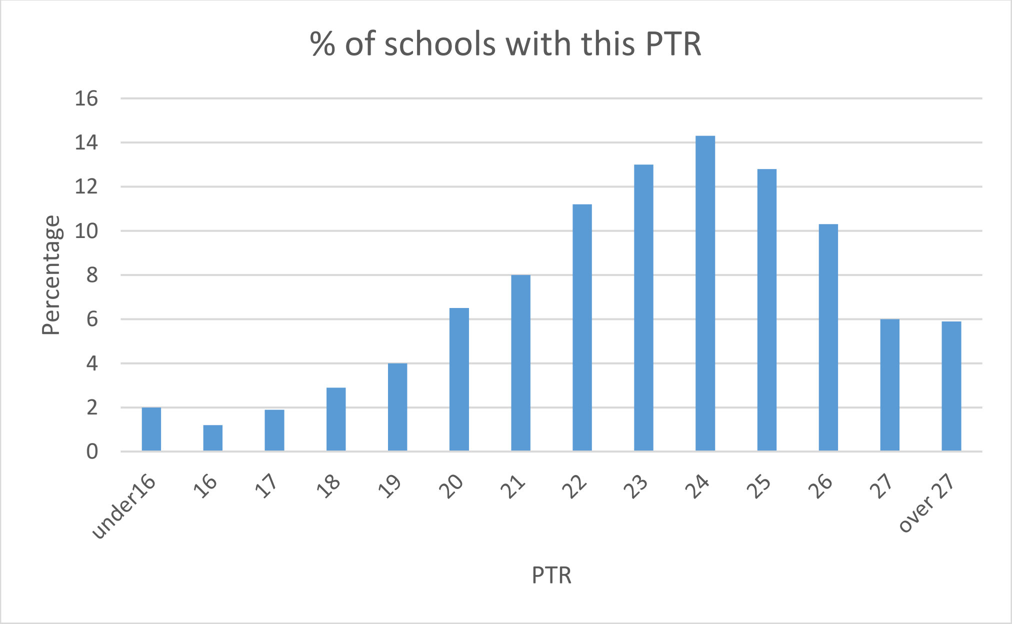
A bar chart shows items represented as vertical or horizontal bars. The length of each bar shows the number of times the item occurs.

Example 1

A survey of primary schools was carried out. The number of pupils and all teachers, including the head teacher, was recorded for each primary school surveyed. The pupil to teacher ratio (PTR) was calculated for each school. The data was recorded in the table shown below.

Pupil teacher ratio (PTR)	<16	16	17	18	19	20	21	22	23	24	25	26	27	>27
% of schools with this PTR	2.0	1.2	1.9	2.9	4.0	6.5	8.0	11.2	13	14.3	12.8	10.3	6	5.9

A bar chart was produced to help the interpretation of the data.



The bar chart shows the data in a visual form.

For each PTR value, a vertical bar has been drawn. The top of the bar represents the corresponding percentage value from the table. For example, the bar for a PTR less than 16 has a height of 2 on the vertical axis, representing 2%, and the bar for a PTR of 23 is represented by 13% on the vertical axis.

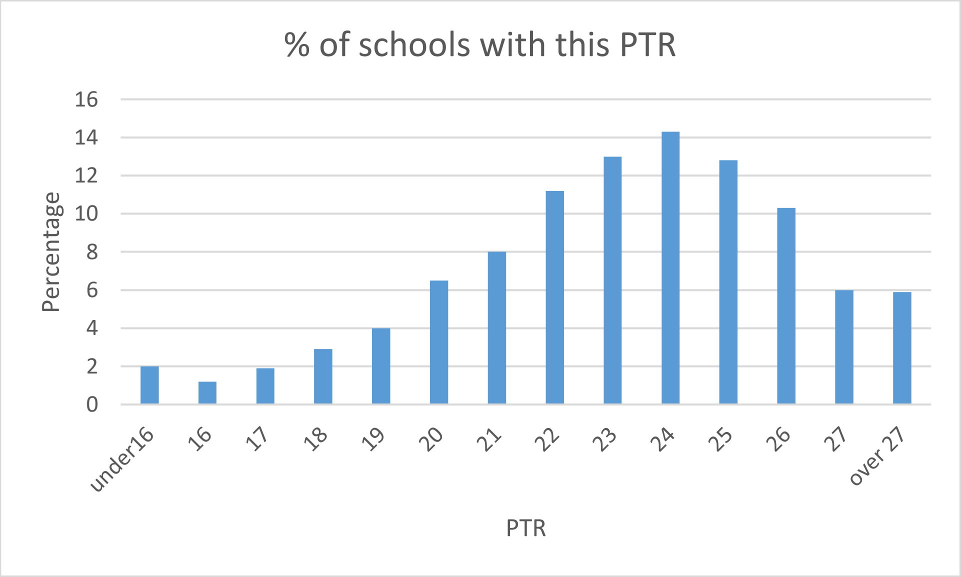
It is often easier to see the pattern in data when it is displayed as a graph rather than in a table. This bar chart shows that most schools have a PTR between 22 and 26 and the most common PTR is 24.

Bar charts are often used in documents containing data, rather than tables. Information must be obtained by using the charts.

Example 2

Use the bar chart to find what percentage of schools have a PTR of 20 or less.

This question can be solved by locating the bars for PTRs of 20 or less, (ie the bars for less than 16, up to and including 20). As the values must be read from the bar chart, some values are likely to be approximate because they cannot be read accurately.



2 + 1.2 + 1.9 + 3 + 4 + 6.5

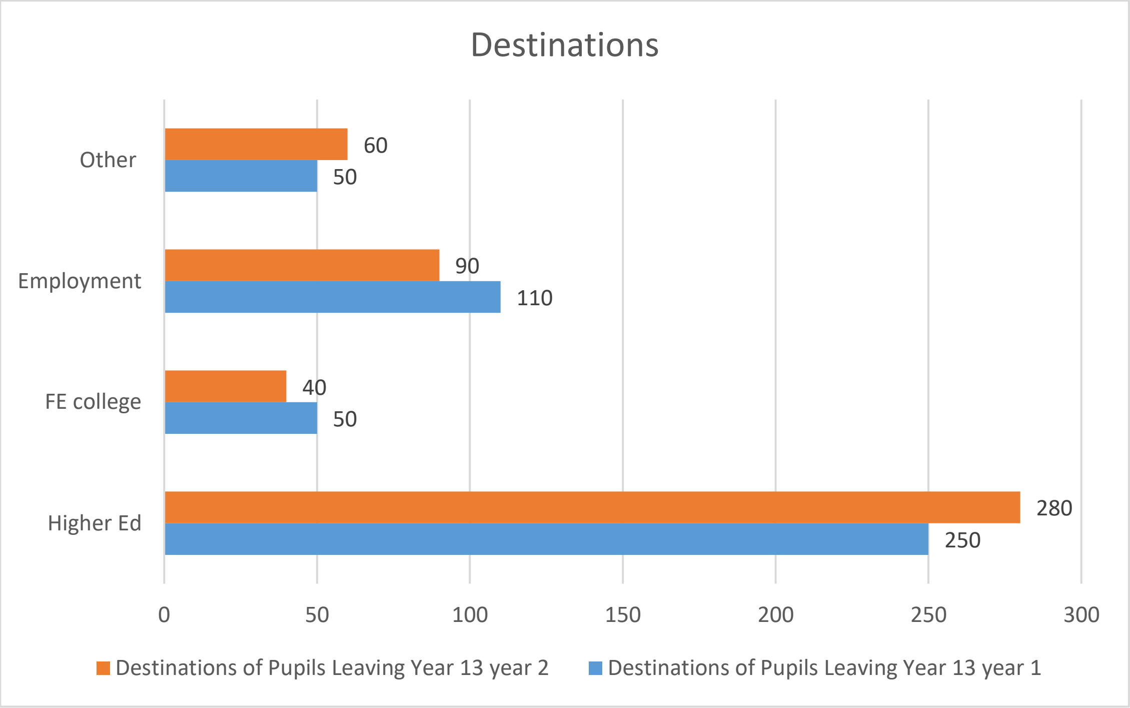
= 18.6%

So, approximately 18.6%of schools have a PTR of 20 or less.

Worked examples

Example 1

A school was tracking which route its pupils took upon leaving year 13, for the next 2 years. How many pupils entered employment in year 1?

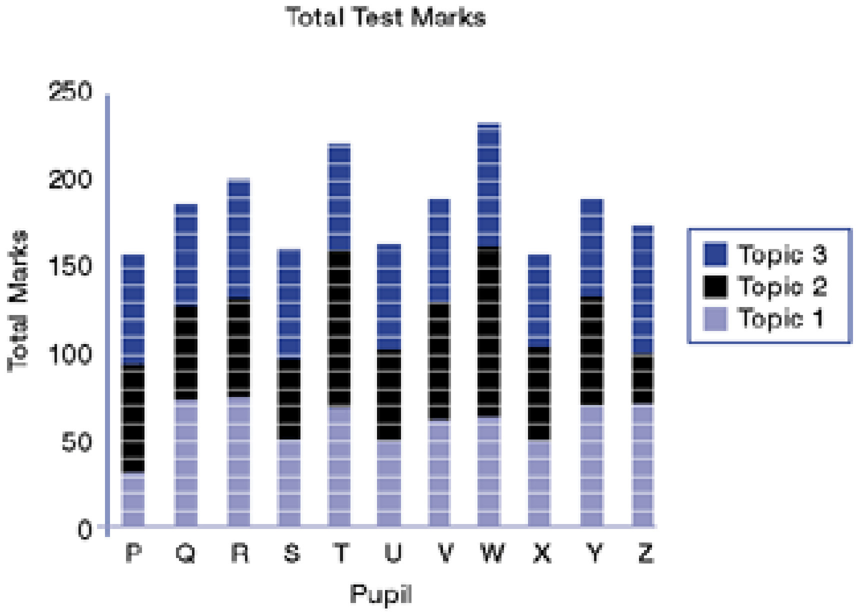


Look at the key to find the appropriate shading for year 1.

Find 'employment' and the end of the shaded bar for year 1. Find the corresponding number on the horizontal ‘number of pupils’ axis.

So, 110 pupils entered employment in year 1.

Example 2



This is a composite bar chart which shows the marks achieved by 11 pupils in 3 tests. Each test covers a different topic from the programme of study. The test result for each topic is shaded and forms part of each bar as shown on the chart.

<u>Which of the following statements are true?</u>

1. Pupil T had the highest total/combined marks for all 3 tests

2. Pupil S had the lowest mark in the test for topic 1 from the 11 pupils

3. None of the pupils scored a total of 200 or more marks

4. Pupil W had the highest score on topic 2 from the 3 tests

Look at each statement in turn and find the information on the graph. Check whether or not the statement is true.

1. The combined total mark is found by looking at the tops of all the combined bars. Pupil T is not the highest bar, so does not have the highest total (Pupil W is highest). This statement is not true.
2. The light blue sections are for topic one. It can be seen that the light blue section for Pupil P is smaller than Pupil S so they achieved a lower mark. The statement is not true.
3. Pupils R, T and W scored 200 or more marks so this statement is not true.
4. Scores on topic 2 can be found by looking at the size of the dark shaded sections of each of the bar. Pupil W has the largest dark shaded section in their bar so has the highest score on this topic. This statement is true.

Pie-charts

A pie chart is a way of illustrating information by using sectors of a circle to represent parts of the whole.

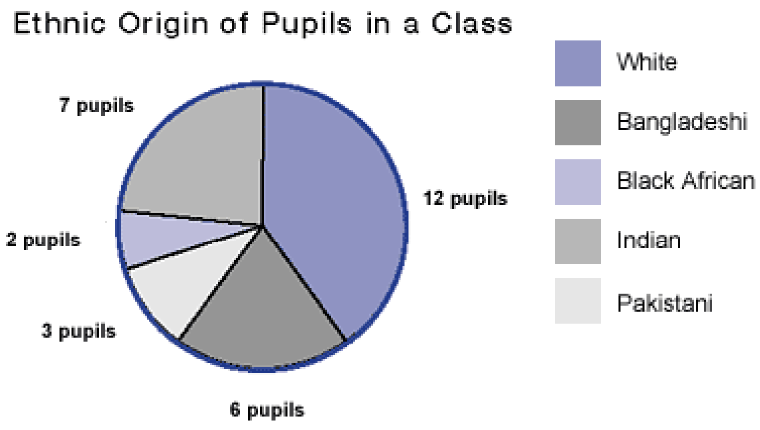
Example

A newly qualified teacher (NQT) was given the following information about the ethnic origins of the pupils in a class.

Ethnic origin	No. of pupils
White	12
Indian	7
Black African	2
Pakistani	3
Bangladeshi	6
Total	30

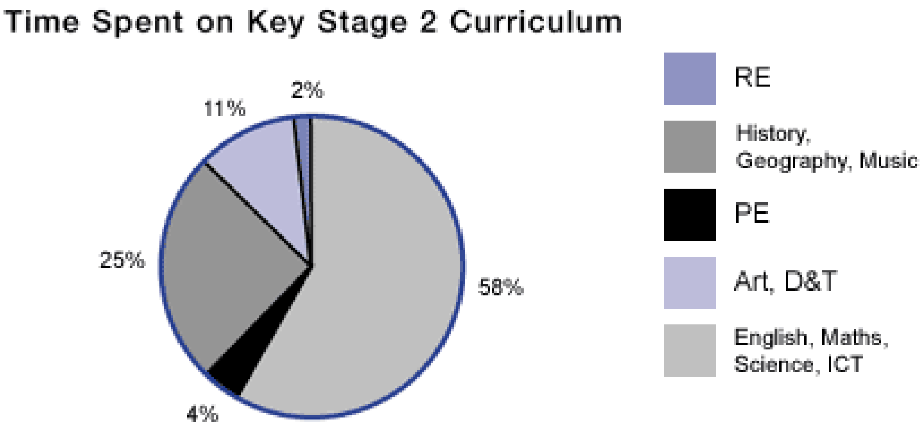
The 30 pupils in the class are classified into 5 different ethnic origins. The whole pie chart represents the class of 30 pupils, and the 5 sectors represent 12, 7, 2, 3 and 6 pupils, as shown in the table.

The pie chart has a legend (key) indicating what it represents.



Worked examples

The pie chart below summarises the amount of time spent on various areas of the key stage 2 curriculum in a primary school.



Example 1

<u>The total time spent each week on the various areas of the curriculum is 26 hours. How much time is spent on history, geography and music?</u>

The pie chart represents 26 hours.

Look at the pie chart and note that the total time spent on history, geography and music is 25%.

25% is ¼, so 25% of the total time is 26 ÷ 4 = 6.5 hours. 6.5 hours = 6 hours and 30 minutes. So 6 hours and 30 minutes is spent on history, geography and music each week.

Example 2

<u>The total time spent each week on the various areas of the curriculum is 26 hours. How much time is spent each week on English, mathematics, science and ICT together?</u>

The pie chart represents 26 hours.

Look at the pie chart and note that 58% of the total time was spent on English, mathematics, science and ICT.

58% can be converted to a decimal.

58% = 58 ÷ 100 = 0.58

Using a calculator, 0.58 x 26 = 15.08

To find how many minutes the 0.08 represents in the answer, 15.08 hours, you can calculate: 0.08 x 60 minutes = 4.8 minutes

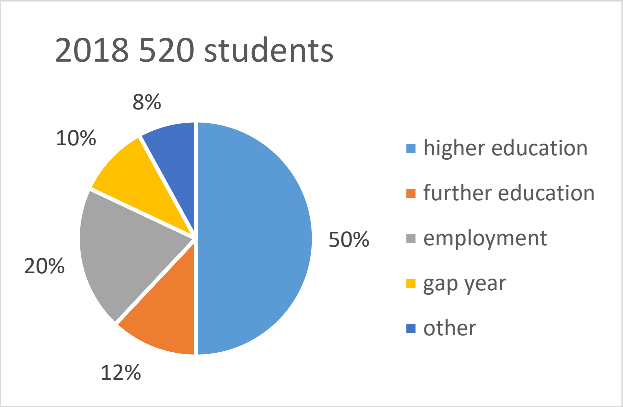
So to the nearest minute, 15 hours and 5 minutes are spent each week on English, mathematics, science and ICT together.

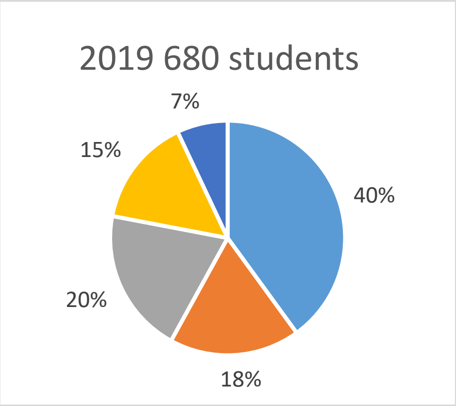
Using pie charts to compare data

Pie charts are often used to show comparisons between 2 sets of data.

Worked example

The following pie charts show an analysis of the destinations of pupils leaving a sixth form college in 2018 and 2019.





Which of the following statements are true?

1. More pupils went into higher education in 2018 than in 2019
2. 136 pupils went into employment in 2019
3. More pupils decided on a gap year in 2019 than in year 2018

Statement 1

In 2018, 50% of pupils went into higher education and in 2019 only 40 %, but the number of pupils is different.
50 % of 520 = 1⁄2 x 520 = 260 pupils
40 % of 680 = 40⁄100 x 680 = 272 pupils
There were more pupils in 2019, so statement 1 is false.

Statement 2

In 2019, 20% of pupils went into employment.
20% of 680 = 20 ÷ 100 x 680 = 136 pupils
Statement 2 is true.

Statement 3

In year 1, 10 % of pupils decided on a gap year. 1⁄100 x 520 = 52 pupils
In year 2, 15 % of pupils decided on a gap year. 15⁄100 x 680 = 102 pupils
Statement 3 is true.

Two-way tables

Two-way tables are designed to allow comparisons between 1 set of data and 2 others. For example, results for GCSE geography and religious education can be compared with the number of pupils obtaining the same grade in GCSE English language.

Example

Tables were prepared to show the relationship between GCSE English language grades and GCSE geography and religious education grades.

	GCSE geography						
	GCSE grade	**7+**	**5-6**	**3-4**	**1-2**	**U-X**	**Total**
GCSE English language	7+	10	6	1			17
	5-6	3	30	16	2		51
	3-4		3	19	7	3	32
	1-2						
	U-X						
	Total	13	39	36	9	3	100

	GCSE religious education						
	GCSE grade	**7+**	**5-6**	**3-4**	**1-2**	**U-X**	**Total**
GCSE English language	7+	6	5				11
	5-6	5	11	1			17
	3-4		12	14	1	1	28
	1-2		1	1	2		4
	U-X						
	Total	11	29	16	3	1	60

Indicate all the true statements:

1. The grade 5+ pass rate for GCSE religious education was exactly 10% higher than the grade 5+ pass rate for geography
2. Of the pupils taking GCSE geography, more than half achieved grade 5 and above in GCSE English language
3. Of the pupils taking GCSE religious education, one third did not achieve grade 5 or above.

Statement 1

Looking at the second table, the religious education results are given in columns. The grades 5+ pass rate for GCSE religious education is 11 + 29 = 40. The total number of pupils taking religious education is 60.

The percentage of pupils who gained a grade 5 or higher was:

$^{40}/_{60}$ x 100% = 66.7%

The grades 5+ pass rate for GCSE geography from the upper table, again shown in the columns, is 13 + 39 = 52. The total number of pupils taking GCSE geography is 100, so the percentage is 52 %.

Statement 1 is therefore false, as 66.7 - 52 is not 10.

Statement 2

Looking at the top table, the results for English language are given in rows. Looking across the top rows, 17 pupils gained grades 7+ for English language and 51 pupils gained grades 5-6. Therefore 68 pupils gained grade 5 and above in English language. 100 pupils took geography and English language.

Therefore statement 2 is true, as 68 is more than half of 100.

Statement 3

Looking at the columns in the second table, the total number of pupils gaining grades 7+ and grades 5-6 is 11 + 29 = 40. A total of 60 pupils took religious education, so 20 pupils did not achieve a grade 5 or above. This is the same as ⅓, so statement 3 is true.

Avoiding common errors

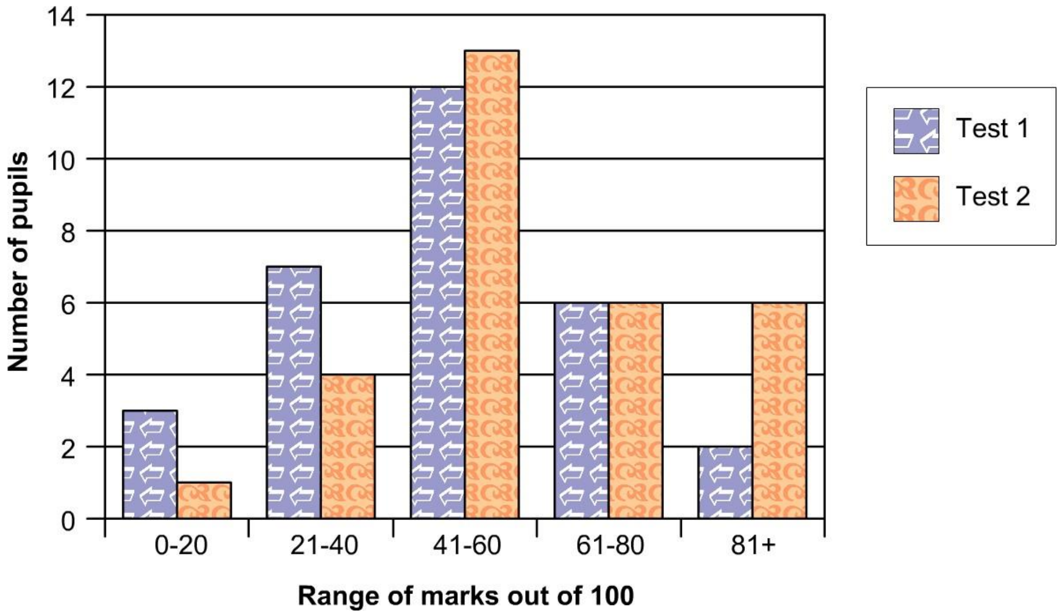
Most common errors can be avoided by:

- using the correct totals;
- using the correct table;

remembering that the totals are always on the opposite side of a table to the category heading.

Now your turn 3.2a

A science class of 30 pupils was given two tests. Test 1 was given at the start of the term and Test 2 at the end of the term. As part of a review of pupil progress, a teacher prepared this bar chart showing pupil achievement in the two tests.



Tick all the true statements:

○ The number of pupils achieving 81+ marks increased by 50% from Test 1 to Test 2.

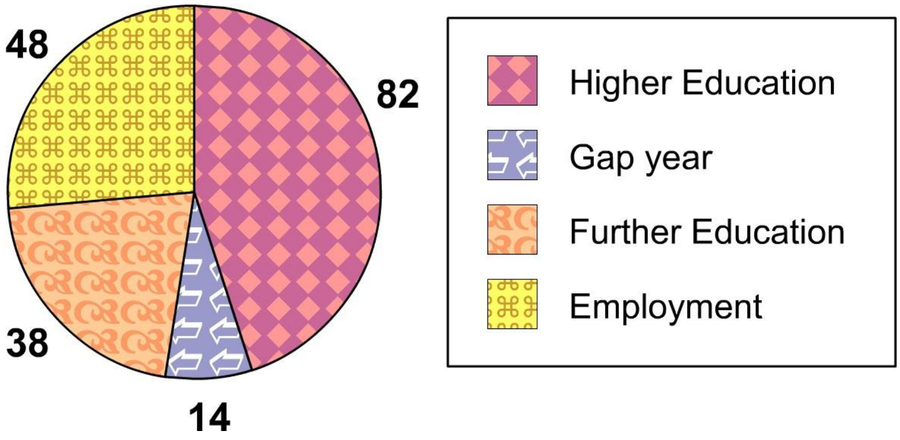
○ More than 80% of pupils achieved more than 40 marks in Test 2.

○ 1/3 achieved fewer than 41 marks in Test 1.

Now your turn 3.2b

The head of careers supplied the following chart showing the destination of Year 13 leavers.

Number of Year 13 students going to different destinations



Tick all the true statements:

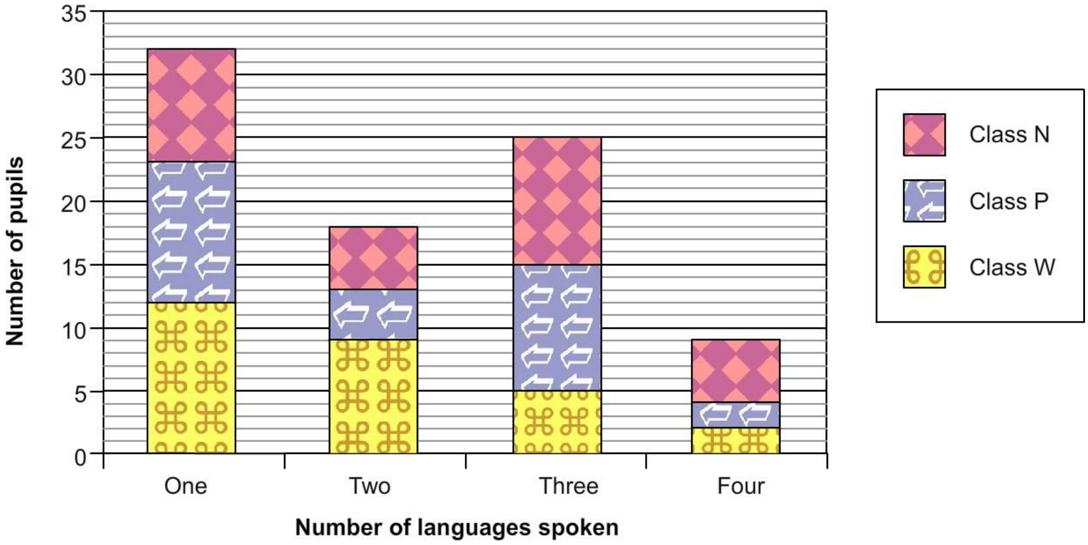
○ 3/7 went on to higher education.

○ 1/13 took a gap year.

○ 2/7 went on to further education

Now your turn 3.2c

In preparation for literacy teaching, a newly appointed Year 2 teacher looked at the number of languages spoken by pupils in the three classes of the year group in the school.



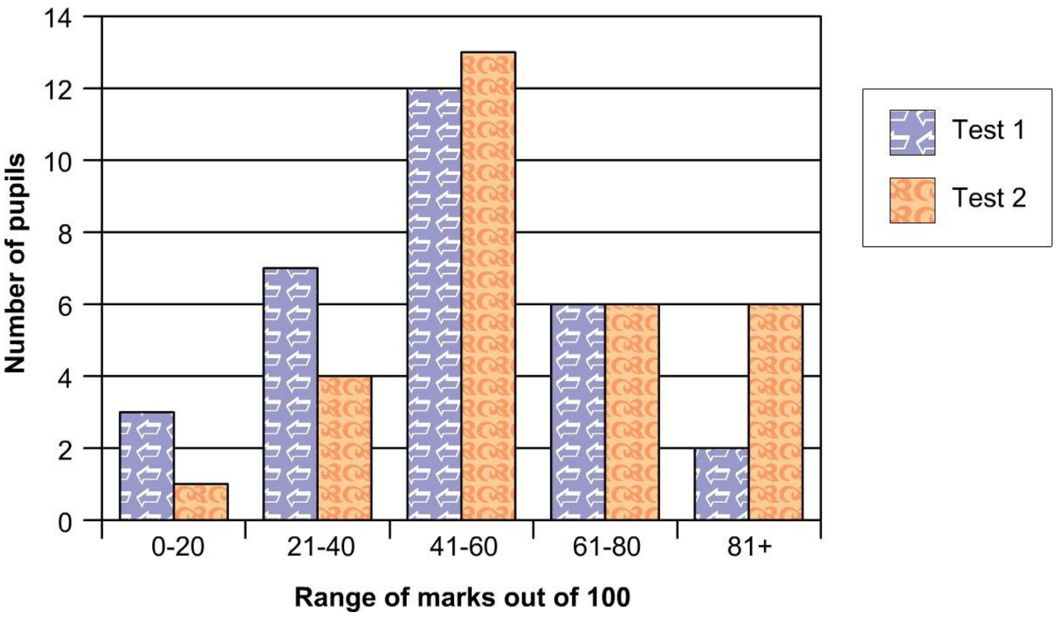
Circle the class in the table which has half of all the pupils in the year group who speak exactly two languages.

Class	Number of Pupils
Class N	29
Class P	27
Class W	28

The answers are on the next page.

Now your turn 3.2a worked solution.

A science class of 30 pupils was given two tests. Test 1 was given at the start of the term and Test 2 at the end of the term. As part of a review of pupil progress, a teacher prepared this bar chart showing pupil achievement in the two tests.



Tick all the true statements:

○ The number of pupils achieving 81+ marks increased by 50% from Test 1 to Test 2.

From the 81+ bar, test 1 = 2 test 2 = 6.
This is an increase of more than 50% (false)

○ ✓ More than 80% of pupils achieved more than 40 marks in Test 2.

41+ = 41-60, 61-80, 81+
13 + 6 + 6 = 25
$\frac{25}{30} \times 100 = 83.3\%$ (true)

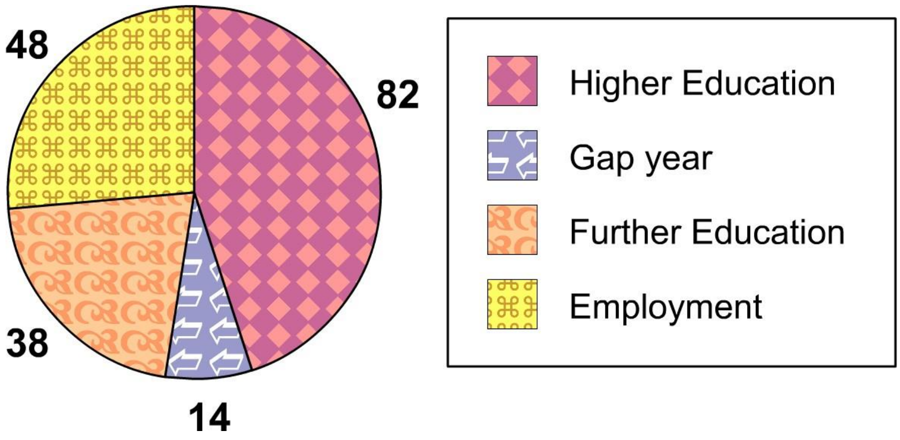
○ ✓ 1/3 achieved fewer than 41 marks in Test 1.

41 or less = 0-20 + 21-40
3 + 7 = 10
$\frac{10}{30} = \frac{1}{3}$ (true)

Now your turn 3.2b worked solution.

The head of careers supplied the following chart showing the destination of Year 13 leavers.

Number of Year 13 students going to different destinations



total students = 48 + 82 + 38 + 14
= 182

Tick all the true statements:

○ 3/7 went on to higher education. $\frac{82}{182} = \frac{41}{91}$ (false)

☑ 1/13 took a gap year. $\frac{14}{182} = \frac{1}{13}$ (true)

○ 2/7 went on to further education. $\frac{38}{182} = \frac{19}{91}$ (false)

Use a scientific calculator to cancel down the fractions.

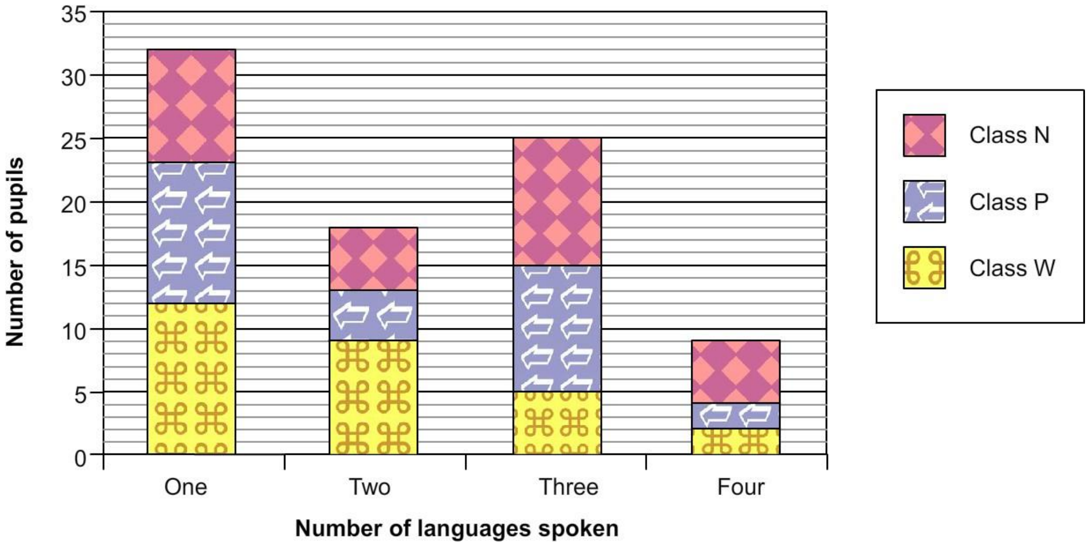
Eg. $\frac{82}{182}$

type in 82÷182 =

to get the answer $\frac{41}{91}$

Now your turn 3.2c worked solution.

In preparation for literacy teaching, a newly appointed Year 2 teacher looked at the number of languages spoken by pupils in the three classes of the year group in the school.



Circle the class in the table which has half of all the pupils in the year group who speak exactly two languages.

Class	Number of Pupils
Class N	29
Class P	27
[✓ Class W]	28

From the 'two' bar 9 out of the total of 18 are from class W.

$\frac{9}{18} = \frac{1}{2}$

3.2 Further learning and support.

Using bar charts https://corbettmaths.com/2012/08/10/reading-bar-charts/

Using pie charts https://corbettmaths.com/2013/05/25/interpreting-pie-charts/

Two-way tables https://www.mathsgenie.co.uk/two-way-tables.html

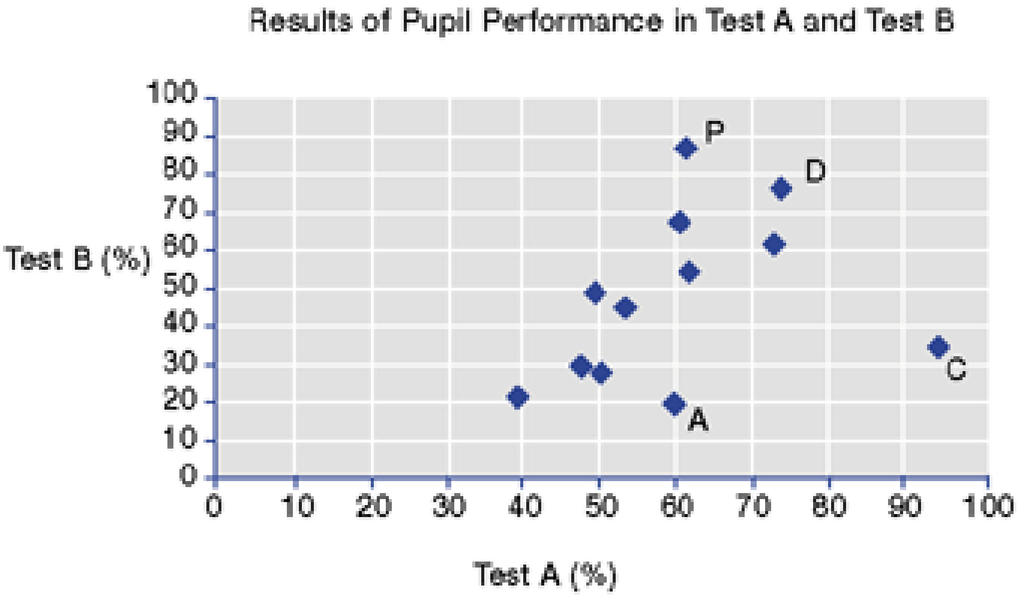
3.3 Interpret and draw conclusions from data relationships presented in a scatter diagram.

Scatter graphs

A scatter graph is a statistical diagram drawn to compare 2 sets of data. It can be used to look for connections or a correlation between the 2 sets of data.

Example

A class took 2 tests. Test A was given early in the course and test B towards the end. The comparative results of these tests are given in the scatter graph below.



Each symbol on the graph shows the scores achieved in both tests by each of the pupils. So, for example, pupil A scored 60% on test A but only 20% on test B.

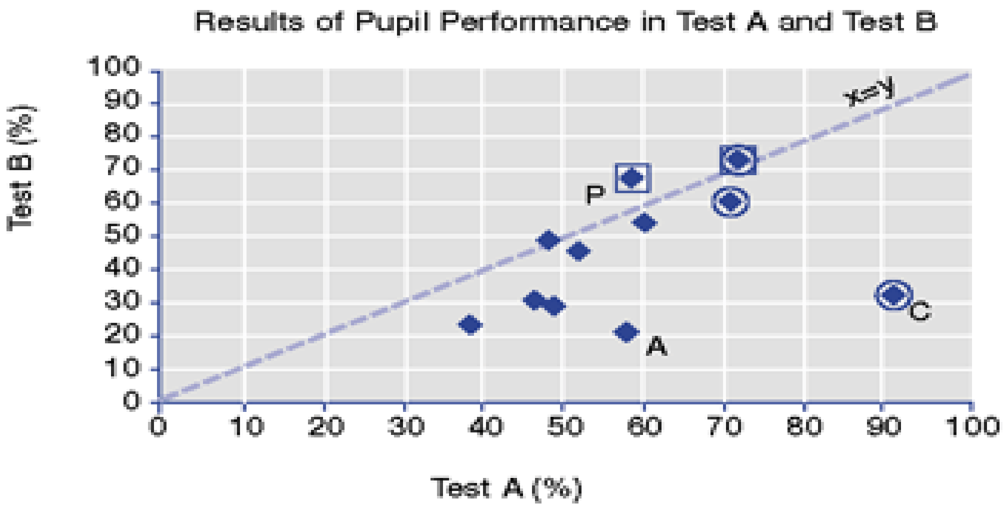
Pupil C achieved the top mark in test A.

Pupil P achieved the top mark in test B.

Pupil marked D came second in both tests.

Making comparisons

To find out whether the pupils generally did better in 1 test or the other, use a ruler or straight edge to draw a line joining marks that are the same on both tests, for example 0 for both, 50 for both, 70 for both, etc.



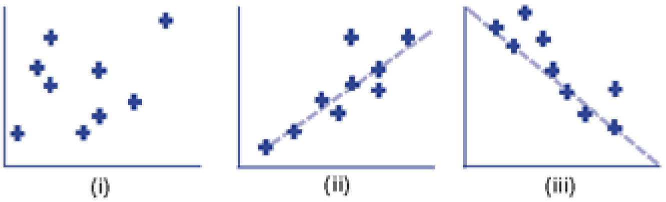
The 2 students above the line achieved higher marks on test B than in test A.

The 8 students below the line achieved higher marks on test A than on test B.

An important use of scatter graphs is to show how 1 set of results relates to another.

If the points on a scatter graph appear to be randomly scattered (see figure (i) below) there is unlikely to be any correlation between the 2 sets of data being measured.

If they form a more regular pattern (as in figure (ii) or figure (iii)), there is likely to be a correlation between them.



The scatter graph for the results from test A and test B, discussed earlier, is similar to figure (ii) and shows a correlation.

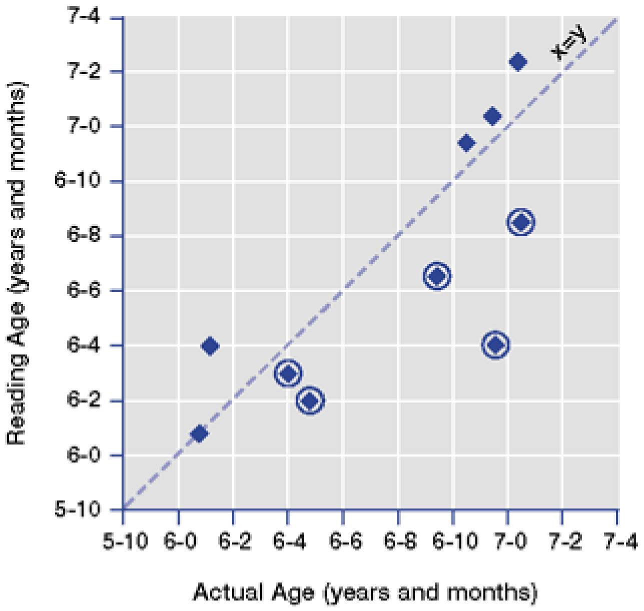
This suggests that test A and test B have a correlation because the pupils performed similarly in both tests. So the tests are likely to be based on the same subject or related subjects and set at a similar level.

The scatter graph enables you to identify particular pupils for whom action might be needed. For example, the reason why C achieved a high mark in test A but a low mark in test B might be investigated.

Worked examples

Example 1

The scatter graph below compares the reading age and the actual age of a group of pupils.



How many pupils have a reading age below their actual age?

To find the answer to this question, it is helpful to draw a line (as shown) through any pair of points that represent the same ages on both scales.

For example, (5-10, 5-10) and (6-8, 6-8) or (7-4, 7-4).

The 5 pupils whose marks are circled have a reading age that is clearly below their actual age.

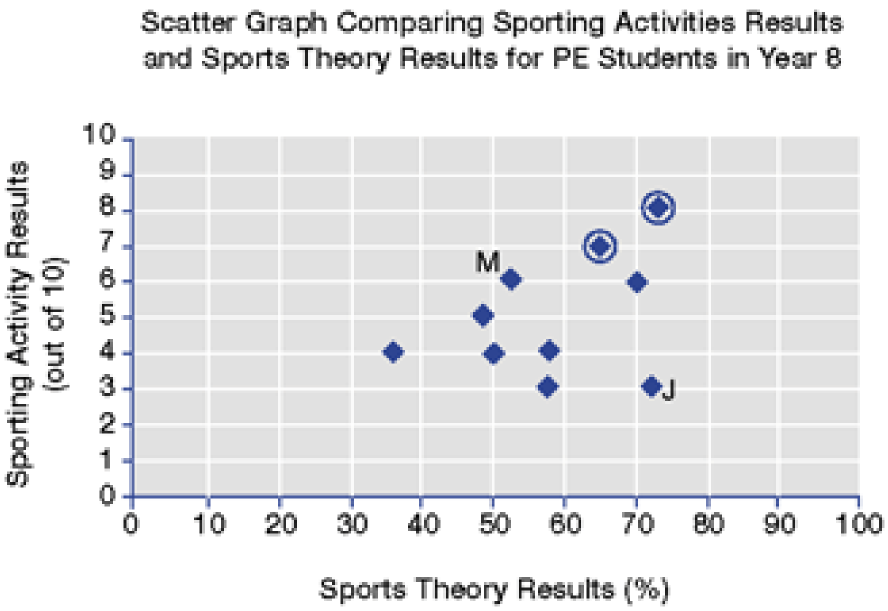
How old is the pupil who has a reading age the same as their actual age?

To find the answer to this question, look at the line you have drawn, joining (5-10, 5-10) and (7-4, 7-4) and find the symbol which lies on it.

The pupil is aged 6 years and 1 month.

Example 2

The PE department has been asked to show whether students in year 8 who do well in sporting activities do equally well in the sports theory part of the course and vice versa.



The students with the higher sporting activity results tend to get higher sports theory results. Those with the top 2 sporting activity marks indicated by circles also are among the top 4 theory results.

However, 1 student, M, achieved quite a high sporting activity result but came seventh out of 10 in sports theory. Conversely, J achieved quite a high mark in sports theory but the lowest mark in sporting activity.

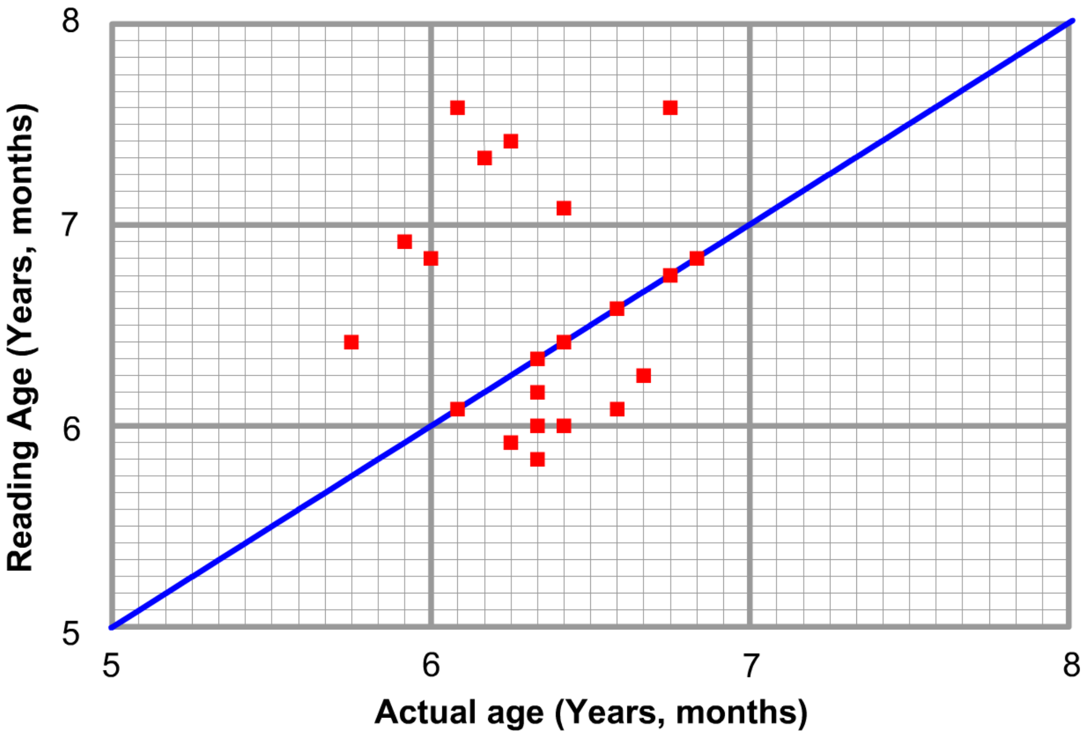
Avoiding common errors

Most common errors can be avoided by:

- using a ruler or straight edge to move across the graph to help locate clusters of points;
- drawing horizontal, vertical or 45 degree lines to break up the graph to allow groups of points to be located more easily; and
- remembering that each point represents 1 person.

Now your turn 3.3

To inform her choice of reading materials, a primary teacher looked at the spread of reading ages in her class. The scatter graph shows
the actual age and reading age of 21 pupils in the class.



3.3a

What proportion of the class have the same reading age as their actual age? Give your answer as a decimal to one decimal place.

3.3b

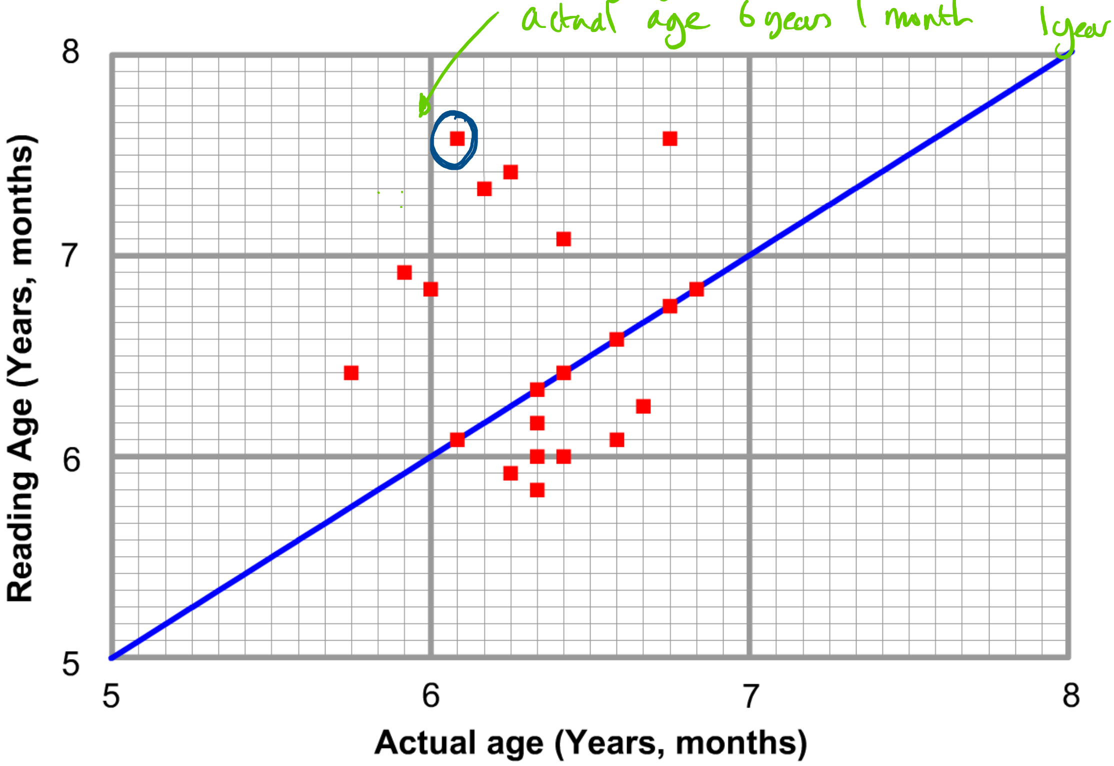
Circle the pupil who has the greatest difference between their reading age and actual age.

3.3c

What is the range of reading ages for the pupils in the class?

The answers are on the next page.

Now your turn 3.3 worked solutions.



reading age 7 years 7 months – difference
actual age 6 years 1 month 1 year 6 months.

3.3a

What proportion of the class have the same reading age as their actual age? Give your answer as a decimal to one decimal place.

Those with the same reading age as their actual age will be on the line. 6 out of the class of 21. $6 \div 21 = 0{\cdot}2857\ldots = 0{\cdot}3$ (1dp)

3.3b

Circle the pupil who has the greatest difference between their reading age and actual age.

This will be the point furthest away from the line (above or below)

3.3c

What is the range of reading ages for the pupils in the class?

The range will be difference between the vertically highest point and the lowest point. 7 years 7 months – 5 years 10 months = 21 months or 1 year 9 months

3.3 Further learning and support

https://www.mathsgenie.co.uk/scatter-graphs.html

3.4 Compare and draw conclusions from two or more data sets presented graphically for example as a box plot or a cumulative frequency diagram

Box and whisker diagram

A box and whisker diagram illustrates the spread of a set of data. It also displays the upper quartile, lower quartile and inter-quartile range of the data set.

A quartile is any 1 of the values which divide the data set into 4 equal parts, so each part represents a quarter of the sample. The upper quartile represents the highest 25%of the data. It can be considered as the median of the upper half of the values in the set.

The lower quartile represents the lowest 25%of the data. It can be considered as the median of the lower half of all the values in the set. The inter-quartile range is the difference in value between the upper quartile and the lower quartile values. The median is the middle value, half of the data set is below and half is above.

Example

You can draw a box and whisker diagram for the results below which were obtained by a year 6 class in an English test marked out of 20.

Pupil A-14
Pupil B-13
Pupil C-3
Pupil D-7
Pupil E-9
Pupil F-12
Pupil G-17
Pupil H 4
Pupil I-9
Pupil J-10
Pupil K-18
Pupil L-16

There are 12 scores. You must place these in order as follows

3, 4, 7, 9, 9, 10, 12, 13, 14, 16, 17, 18.

The range is 18 - 3 = 15.

There is an even number of values so the median is the midway between 10 and 12. The median has the value 11.

The upper quartile is the median value for 12, 13, 14, 16, 17, 18 and so is midway between 14 and 16. The upper quartile is 15. So 25% of all the pupils score above 15 and 75%score less than 15.

The lower quartile is the median value for 3, 4, 7, 9, 9, 10 and so is midway between 7 and 9. The lower quartile is 8. So 25%of all the pupils score less than 8 and 75% score more than 8.

Had there been an odd number of values, the median would be the middle value, but in determining the lower and upper quartiles the median value is then ignored. For example, if the values are:

2, 3, 5, 7, 9, 13, 15, 18, 19, 20, 22, 23

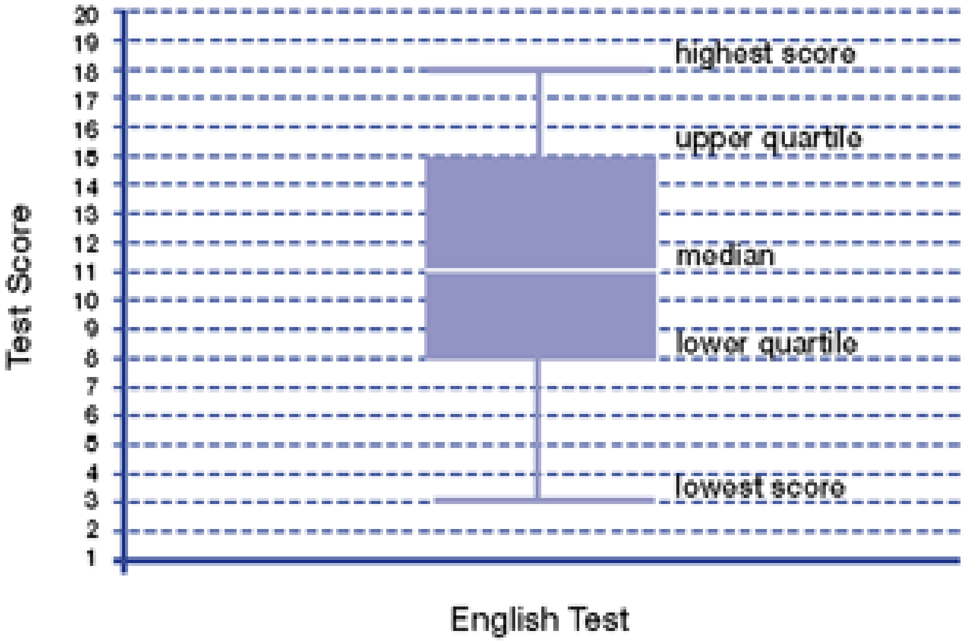
The median value is 14.

A box and whisker diagram can be used to display this information. The table below shows the results of an English test, constructed for presentation as a box and whisker diagram.

English test scores out of 20

Lowest test score	3
Highest test score	18
Lower quartile	8
Upper quartile	15

The data in the table is represented by a box and whisker diagram as follows:



A box and whisker diagram shows at a glance the range of scores of the middle 50% of pupils (the box) and the total range of all the scores (the whiskers).

A box and whisker diagram shows at a glance the range of scores of the middle 50%of pupils (the box) and the total range of all the scores (the whiskers).

The 2 whiskers show the highest and lowest values from which the range of the data set can be calculated. For this data set, the range is given by 18 - 3 = 15.

The box gives the range of the middle 50% normally called the inter-quartile range. It represents the range of the scores between 25% (the lower quartile) and 75% (the upper quartile), hence the middle 50%. In this example, the inter-quartile range is given by 15 - 8 = 7. Comparisons between different sets of data (eg different test scores) can be made by plotting a box and whisker diagram for each set of scores on the same graph.

Box and whisker plots can be constructed using cumulative frequency curves or cumulative percentage curves as these curves can be used to show the quartiles and the median. You should also read the section on cumulative frequency graphs.

Worked examples

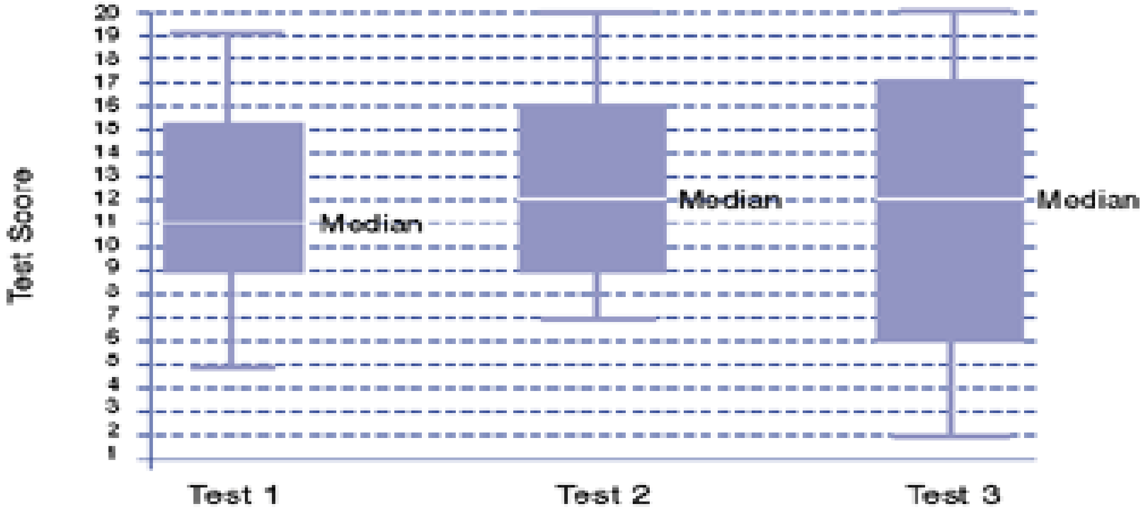
Example 1

A year 9 class completed 3 science tests. The scores for the 3 tests are presented in the table below. Display this data using a box and whisker diagram.

<u>Year 9 science test scores - marks out of 20</u>

	Test 1	**Test 2**	**Test 3**
Lowest score	5	7	2
Highest score	19	20	20
Lower quartile	9	9	6
Upper quartile	15	16	17

The box and whisker diagrams for each test are shown below:



The diagrams show that in test 3 pupils achieved a wider spread of results (from 2 to 20 with a range of 18) than in tests 1 and 2. This could lead teachers to ask questions about why pupils' performance ranged more widely on this test than on the others. In the diagram for test 1, the 2 ‘whiskers’ are the same length, so the upper and lower quartile ranges are the same.

Example 2

The scores for 3 tests completed by a year 5 group are:

	Test 1 (score out of 20)	**Test 2 (score out of 50)**	**Test 3 (score out of 100)**
Lowest score	5	12	20
Highest score	19	43	75
Lower quartile	9	19	35
Upper quartile	15	35	70

Which of the following statements are true?

1. The highest and lowest marks for the year group declined over the 3 tests
2. There was no change in the year group's performance over the 3 tests
3. Marks for the year group improved over the 3 tests

All the tests have different maximum scores and will be easier to compare if converted into percentages. Box and whisker diagrams can be drawn on the same scale and comparisons made between the tests.

Test 1 scores are out of 20 and need to be converted so that they are out of 100, by multiplying top and bottom of the fraction by 5 (because 5 x 20 = 100).

$\frac{5}{20}$	=	$\frac{25}{100}$	so	$\frac{5}{20}$	= 25%
$\frac{19}{20}$	=	$\frac{19 \times 5}{100}$	so	$\frac{19}{20}$	= 95%
$\frac{9}{20}$	=	$\frac{9 \times 5}{100}$	so	$\frac{9}{20}$	= 45%
$\frac{15}{20}$	=	$\frac{15 \times 5}{100}$	so	$\frac{15}{20}$	= 75%

Test 2 scores are out of 50, so can be converted by doubling each score.

12 50	=	24 100	= 24%
43 50	=	86 100	= 86%
19 50	=	38 100	= 38%
35 50	=	70 100	= 70%

Test 3 scores are already in percentages, because they are already given out of 100.

The figures show that both the highest and lowest scores have fallen from test 1 to test 2, and from test 2 to test 3. As statement 1 is true, statements 2 and 3 must be false.

Cumulative frequency

A cumulative frequency graph shows the cumulative totals of a set of values up to each of the points on the graph.

Example

A teacher arranged the marks gained by all year 10 pupils in a mathematics test in a table as shown below:

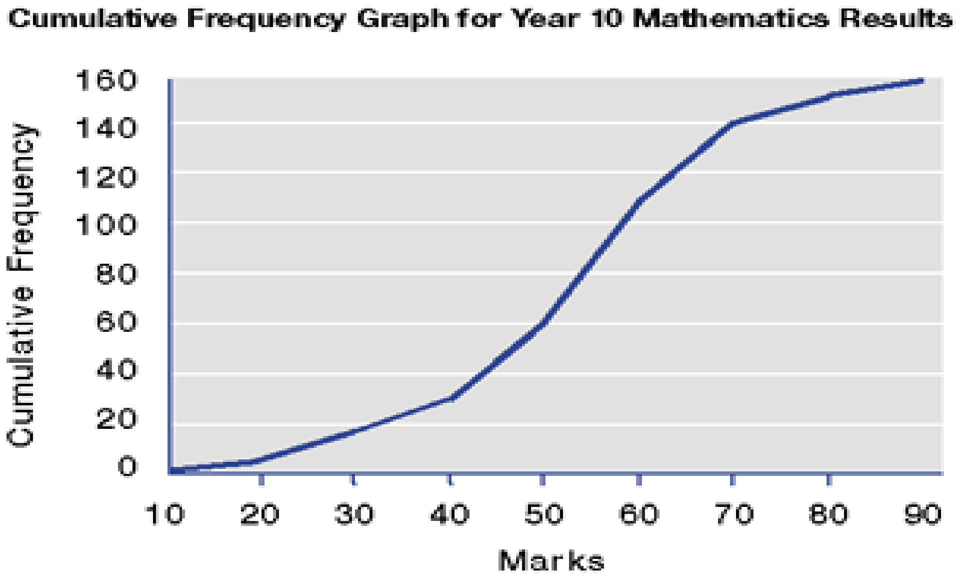
Marks	Frequency of pupils
11-20	2
21-30	11
31-40	19
41-50	36
51-60	42
61-70	31
71-80	13
81-90	6

This table shows the number of pupils (called the frequency) who gained marks in the various mark bands (eg 31 to 40). For example, the number of pupils who scored between 21 and 30 marks was 11. No pupil scored fewer than 11 marks or more than 90 marks.

To create a cumulative total for the frequency of pupils in each group (called the cumulative frequency) a third column is created as shown below:

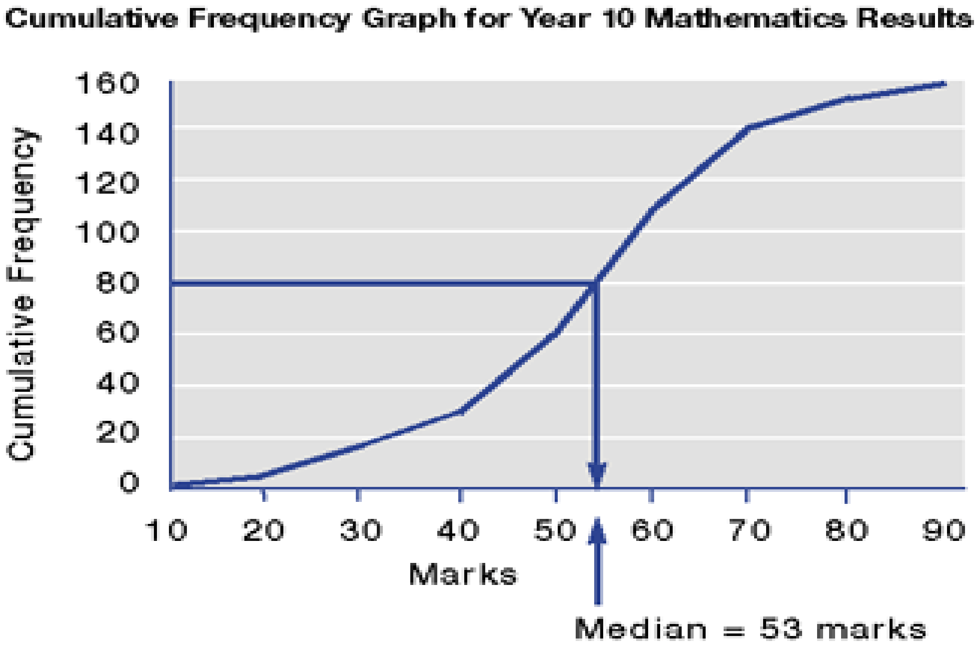
Marks	Frequency	Cumulative total	Cumulative frequency
11-20	2	2	2
21-30	11	2+11	13
31-40	19	13+19	32
41-50	36	32+36	68
51-60	42	68+42	110
61-70	31	110+31	141
71-80	13	141+13	154
81-90	6	154+6	160

The cumulative frequency column makes it easy to see at a glance that 68 pupils scored 50 marks or fewer, and that 32 pupils scored 40 marks or fewer.



A cumulative frequency graph is a way of presenting information visually, which allows other information to be deduced.

For example, from the graph we can obtain the median (or middle) mark. The median is the mark which half of all pupils exceed and half do not reach. As there are 160 pupils, we need to find 80 pupils, and then draw a line across until it meets the graph. Drawing a vertical line down from this point and reading the number of marks at that point shows that the median is 53 marks.



It is also possible to find the upper and lower quartile marks from the graph.

In this example, the lower quartile is the mark which 1 quarter of all pupils' scores does not reach.

To find the lower quartile, find the point 1 quarter of the way up the vertical axis, which is 40 on the cumulative frequency axis. Draw a line across from the 40 mark until it meets the graph. Draw a vertical line down from this point. The lower quartile is 43 marks. Therefore a quarter of the marks lie below 43 marks.

To find the upper quartile, we need to find the point that is three quarters of the way up the vertical axis which is 120 on the cumulative frequency axis. Draw a line across from the 120 mark until it meets the graph. Draw a vertical line down from this point. An estimate of the upper quartile is 63. Therefore three-quarters of the marks lie below 63 marks.

The inter-quartile range is often used to give an idea of how widely the items of data are spread out.

The inter-quartile range is found by calculating the difference in value between the upper quartile and the lower quartile, that is:

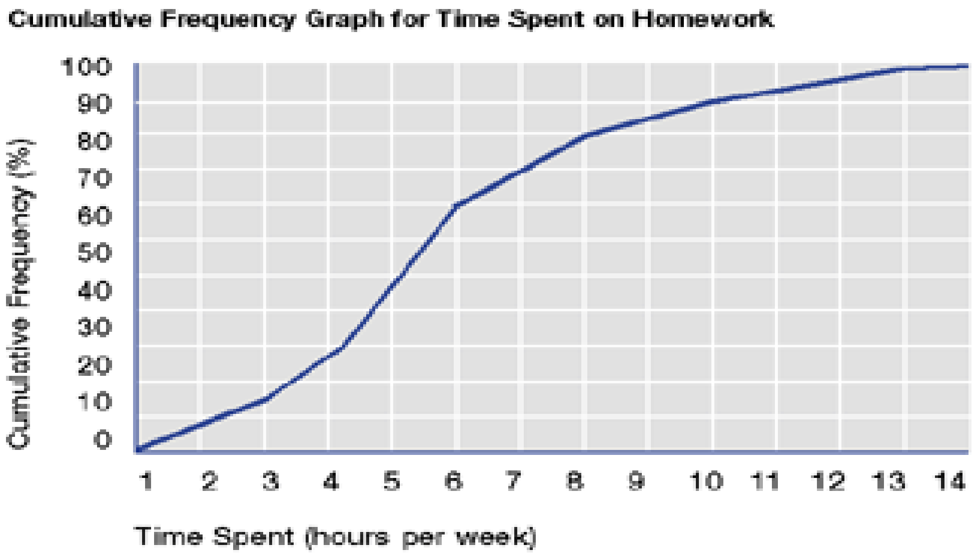
upper quartile value - lower quartile value.

In this example, the estimate of the inter-quartile range is 63 - 43 = 20.

The marks of the middle 50 per cent of pupils lie roughly between the lower quartile mark, 43, and the upper quartile mark, 63.

If this information was shown using a box and whisker diagram, the box would be drawn with the left-hand edge at 43 and the right-hand edge at 63.

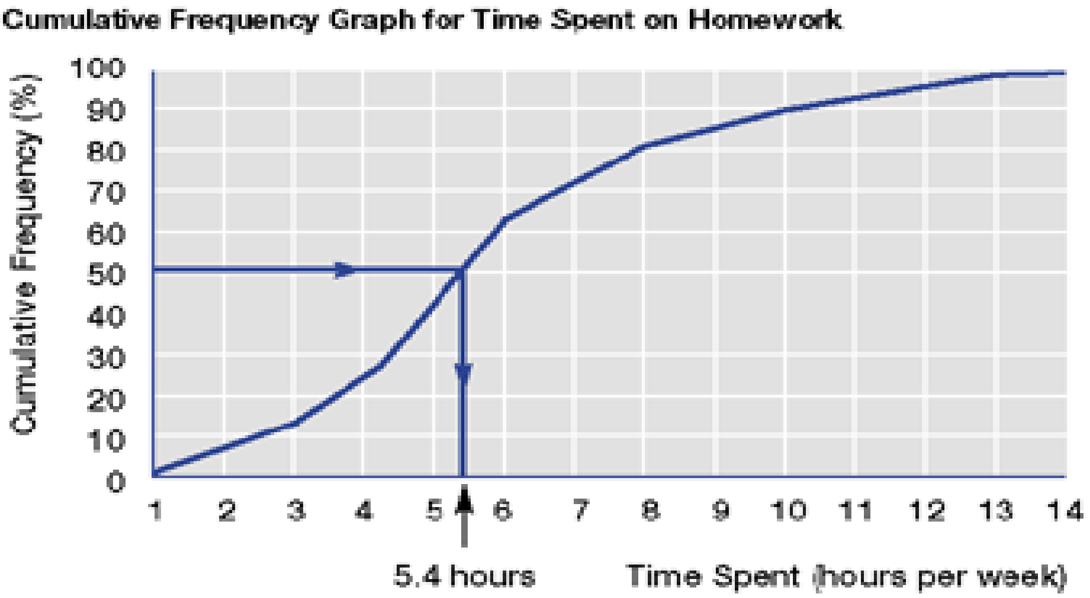
Worked examples



Example 1

What is the median time that is spent on homework?

The median value is the number of hours spent on homework by the middle pupil(s). The middle value will be at 50%as the graph shows values from 0-100 %.



From this point read across to the graph and draw a vertical line down to the horizontal axis, giving an estimate of the median as 5.4 hours.

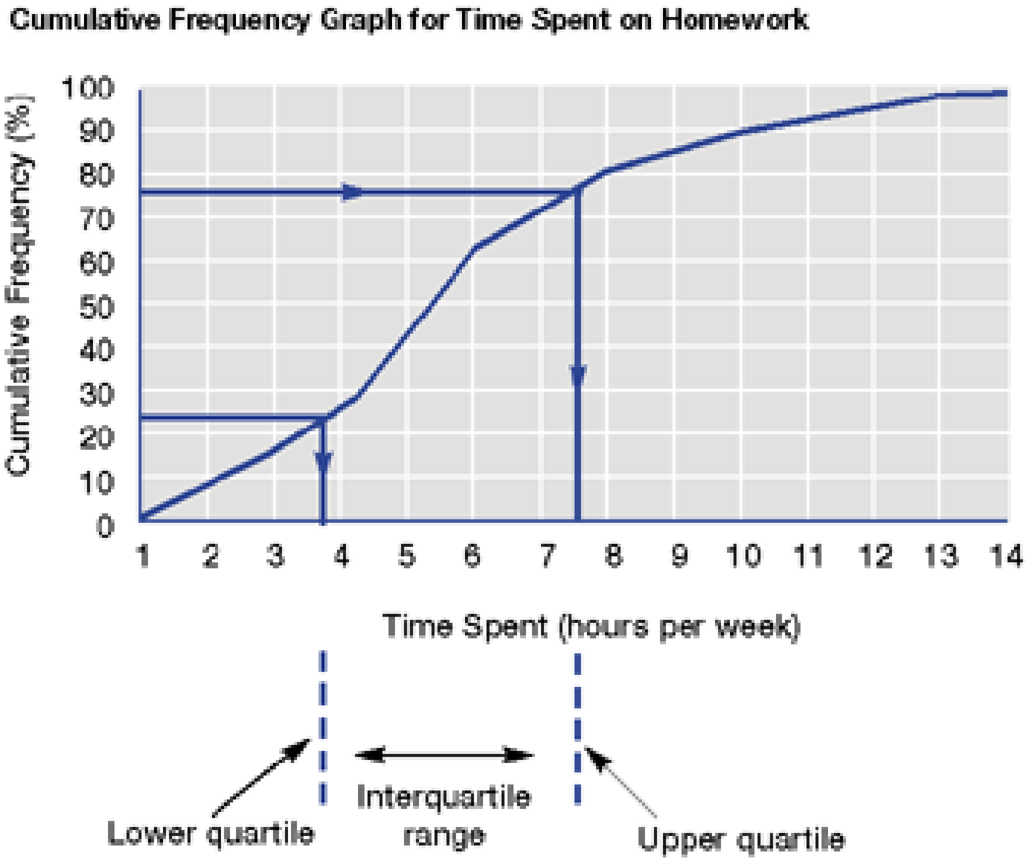
The median is 5.4 hours, or 5 hours 24 minutes.

Example 2

<u>What is the inter-quartile range?</u>

It is necessary to first find the upper and lower quartiles.

To find the lower quartile find the number of hours of homework done by 25 % or fewer of the pupils.



Reading across from 25 % and down from the graph gives an estimate of the lower quartile as just under 4 hours.

Following the same process, the upper quartile at 75 % is about 7½ hours.

From this an estimate of the inter-quartile range is found as:

upper quartile - lower quartile

= 7½ hours

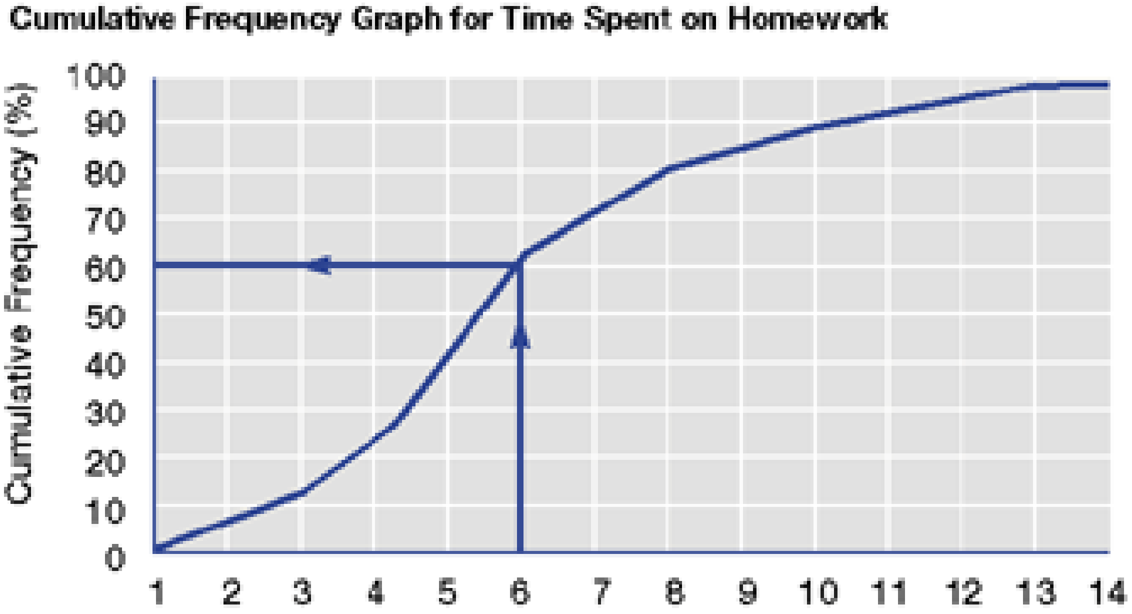
So the inter-quartile range is roughly 3½ hours.

The number of hours spent on homework by the middle 50 % of the pupils ranges from 4 hours to 7½ hours.

Example 3

What percentage of pupils spends 6 hours or less on homework per week?

Find the point marked '6 hours' on the horizontal axis. Move from the axis vertically up to the curve. Move from the curve horizontally to the vertical axis. Read the value from the graph. In this case the value is 60.

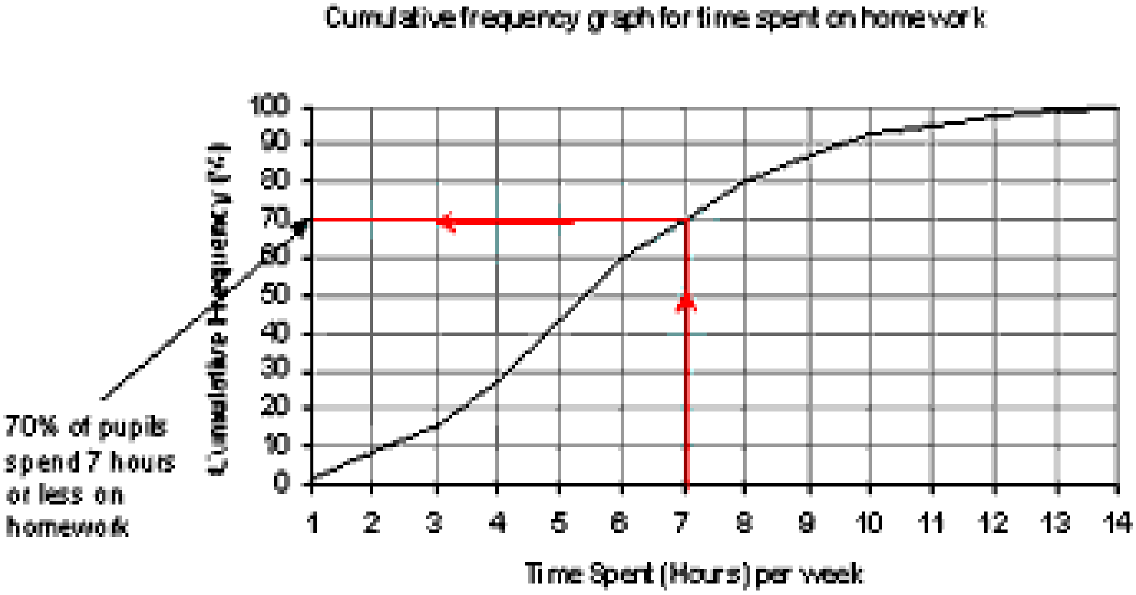


Therefore about 60 per cent of the pupils spend six hours or less on homework.

Example 4

<u>What percentage of pupils spends more than 7 hours on homework?</u>

Find the point marked '7 hours' on the horizontal axis and read the corresponding value from the graph. This shows that 70 % of pupils spend 7 hours or less on homework.



The question asked was how many pupils spend more than 7 hours per week on homework, therefore 100 - 70% = 30% spend more than 7 hours on homework.

70% of pupils spend 7 hours or less on homework per week.

So 30% of pupils spend more than 7 hours on homework per week.

Note: percentages are sometimes referred to as percentiles in this context. Percentiles are defined in the glossary.

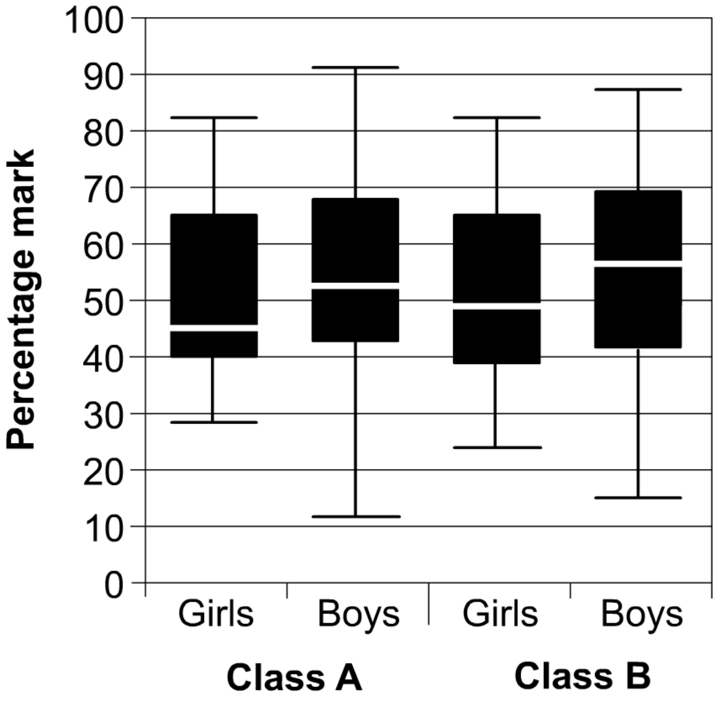
Avoiding common errors

- Ensure the cumulative frequency graph is always plotted using the correct axes (cumulative frequency is always plotted on the vertical axis).
- Decide whether a question is asking for ‘more than’ or ‘less than’ a particular value (see example 4).
- Decide whether to read from the vertical axis to the horizontal axis or vice versa.

Now your turn 3.4a

At the departmental meeting, the Head of Mathematics used a box-and-whisker diagram to show pupils' percentage

mock GCSE examination marks for boys and girls in two classes.



Tick all the true statements:

☐ Class A boys have the greatest range of percentage marks.

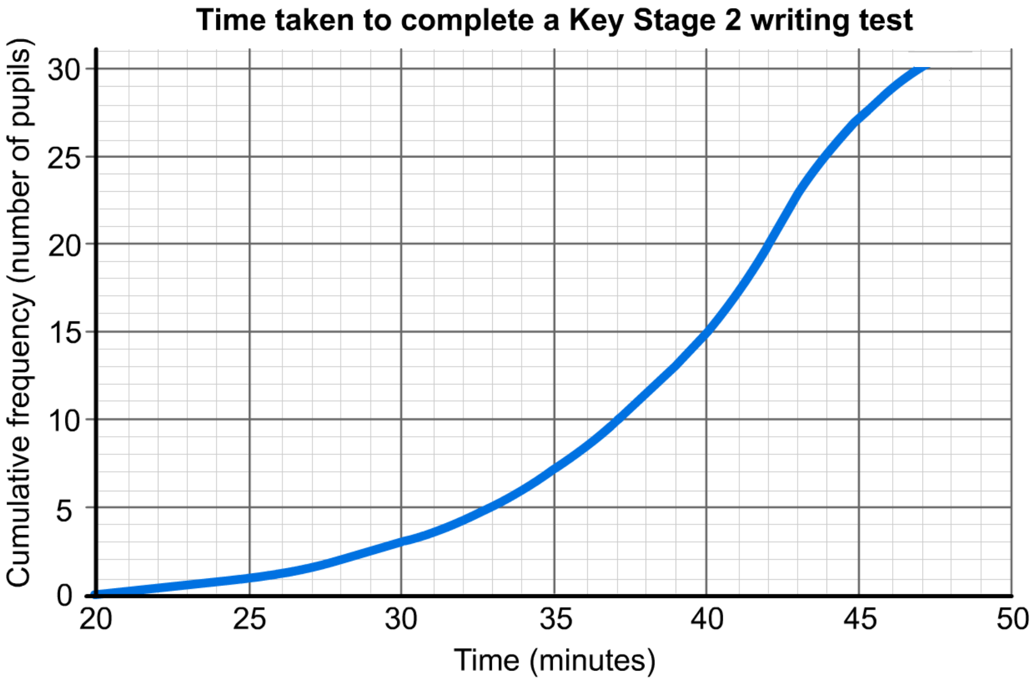
☐ Class A girls have the lowest median and the smallest range of percentage marks.

☐ Only Class B boys have a median percentage mark above 50%.

Now your turn 3.4b

A Year 6 primary teacher asks pupils to record the finish time, to the nearest minute, for a practice Key Stage 2 writing test. Thirty pupils do the test.

The maximum time allowed for the test is 50 minutes. To inform a staff discussion on pupils' performance in the test, the teacher prepares a cumulative frequency graph showing the time pupils took to complete the test.



Tick all the true statements:

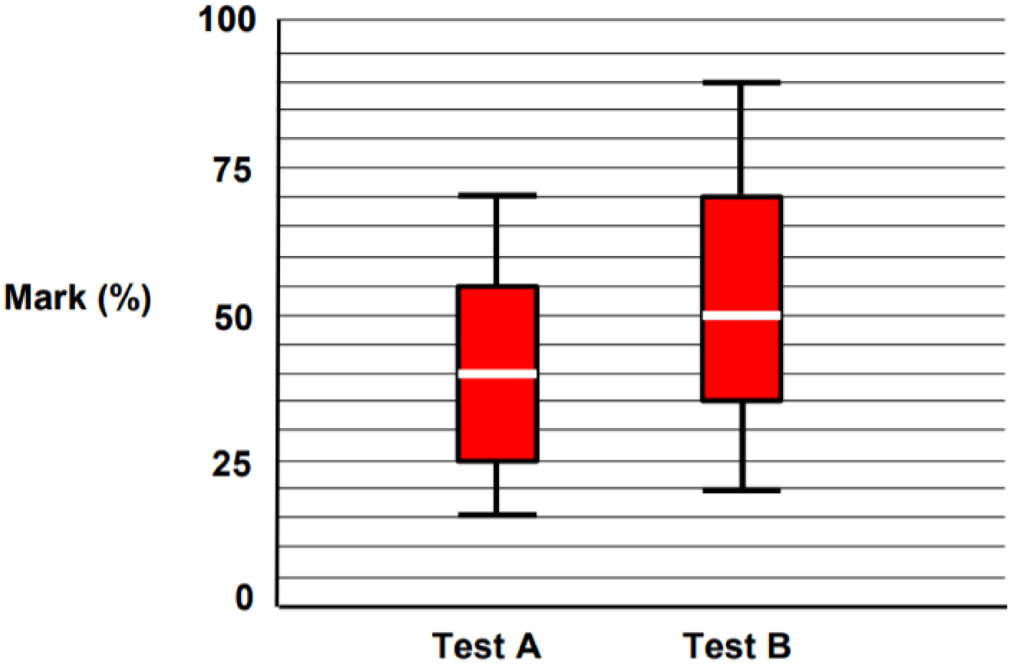
☐ All the pupils completed the test within the maximum time allowed.

☐ The median time taken was 40 minutes.

☐ No pupils recorded a time less than 29 minutes.

Now your turn 3.4c

A geography teacher uses a box-and-whisker graph to compare the performance of his class in two tests.



Tick all the true statements:

☐ The range of marks in Test A was greater than in Test B.

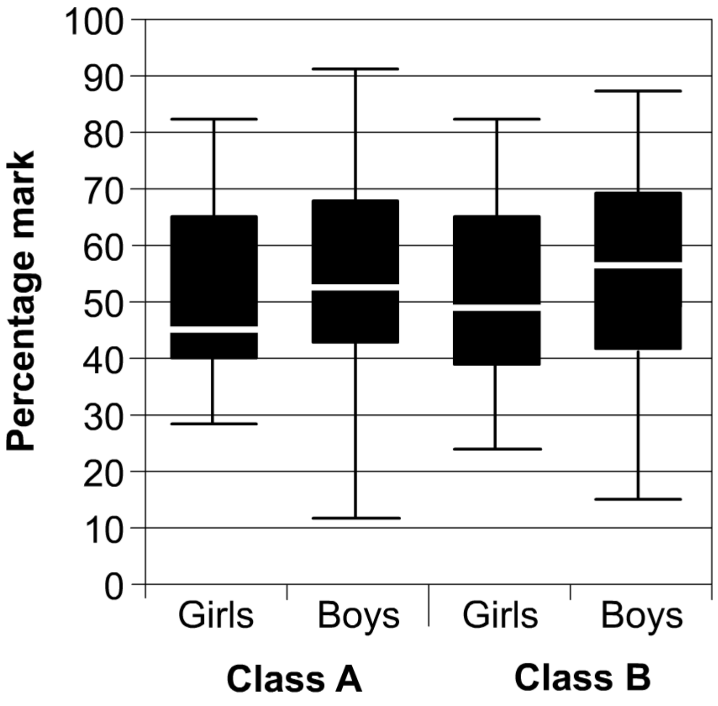
☐ The median mark in Test B was approximately 10 percentage points higher than the median mark in Test A.

☐ In Test B one-quarter of the pupils achieved 75% or more.

The answers are on the next page.

Now your turn 3.4a worked solution.

At the departmental meeting, the Head of Mathematics used a box-and-whisker diagram to show pupils' percentage
mock GCSE examination marks for boys and girls in two classes.



Tick all the true statements:

☑ Class A boys have the greatest range of percentage marks.

The largest range will have the greatest distance between the ends of the whiskers. True.

☑ Class A girls have the lowest median and the smallest range of percentage marks.

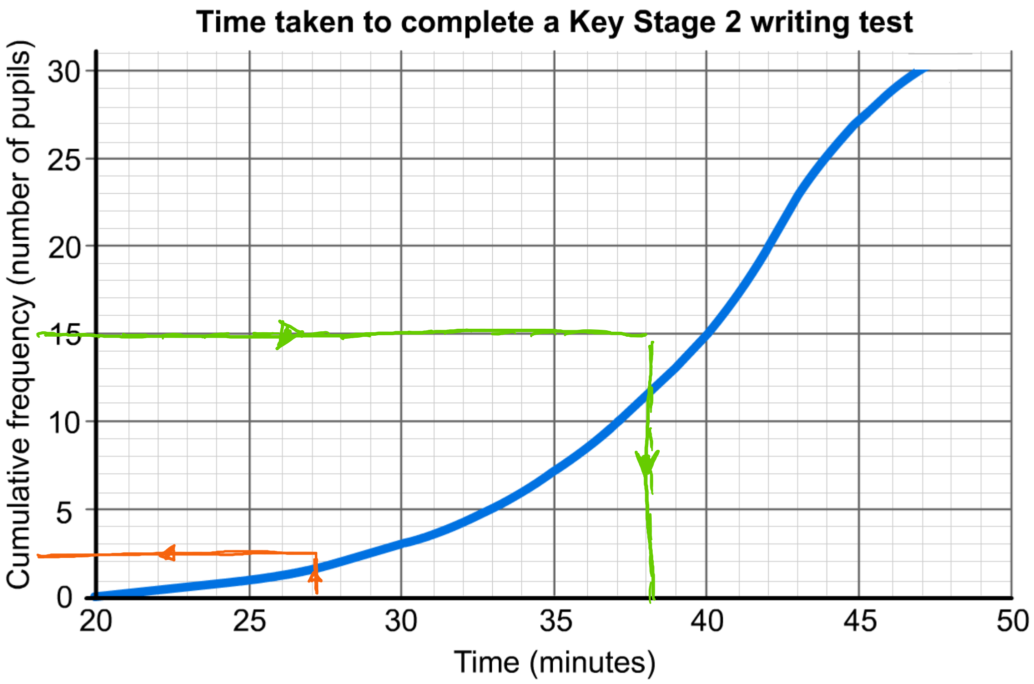
The lowest median (white line) is Class A girls. The smallest distance between the ends of the 'whiskers' is also Class A girls. True

☐ Only Class B boys have a median percentage mark above 50%.

Both class A boys and class B boys have median above 50%. False

Now your turn 3.4b worked solution.

A Year 6 primary teacher asks pupils to record the finish time, to the nearest minute, for a practice Key Stage 2 writing test. Thirty pupils do the test.
The maximum time allowed for the test is 50 minutes. To inform a staff discussion on pupils' performance in the test, the teacher prepares a cumulative frequency graph showing the time pupils took to complete the test.



Tick all the true statements:

☑ All the pupils completed the test within the maximum time allowed.
All 30 finished in 47 minutes. True.

☑ The median time taken was 40 minutes.
30÷2=15 The 15th student will be the median time. See graph. True

☐ No pupils recorded a time less than 29 minutes.
Reading up from 29 minutes, at least 2 students took less than 29 minutes. See graph. False

Now your turn 3.4c worked solution.

A geography teacher uses a box-and-whisker graph to compare the performance of his class in two tests.

Tick all the true statements:

☐ The range of marks in Test A was greater than in Test B.

A: 70 - 15 = 55   B: 90 - 20 = 70   False

Range is the distance between the ends of the 'whiskers'

☑ The median mark in Test B was approximately 10 percentage points higher than the median mark in Test A.

true

A: median ≈ 40%   B: median ≈ 50%.   50-40 = 10%

medians are the white lines in the box

≈ - means approximately equal

☐ In Test B one-quarter of the pupils achieved 75% or more.

The higher whisker represents the top quarter. It starts at 70%. False.

3.4 Further learning and support

Box plots https://www.mathsgenie.co.uk/box-plots.html and https://www.drfrostmaths.com/videos.php?skid=388

Cumulative frequency https://www.mathsgenie.co.uk/cumulative.html and https://www.drfrostmaths.com/videos.php?skid=387

3.5 Identify patterns or trends within data sets presented graphically such as a line graph or a time series graph.

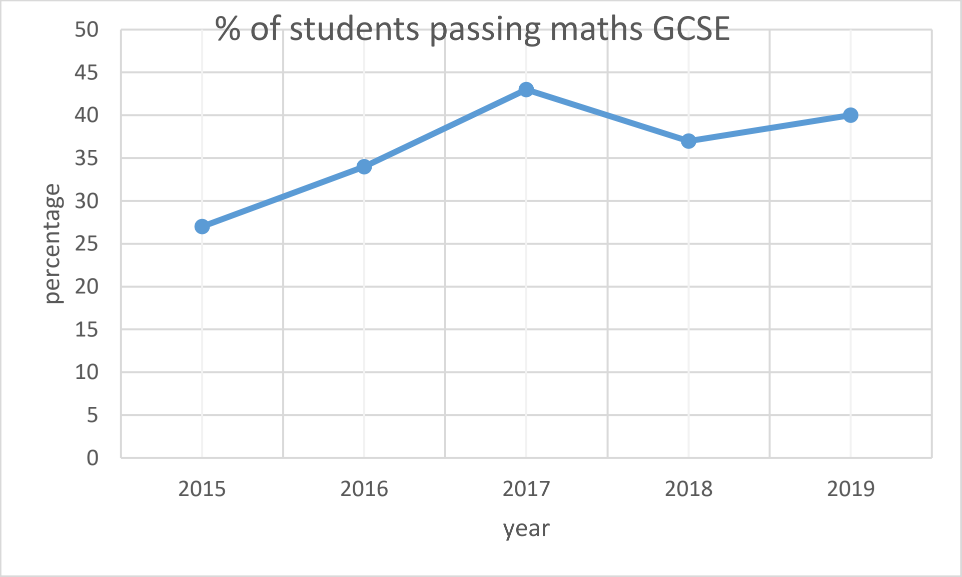
Line graphs

A line graph is a visual representation of two sets of related data. It is the name given to a graph where the individual points are joined by a line or lines. If the horizontal axis represents time, it is sometimes known as a time series graph.

Example

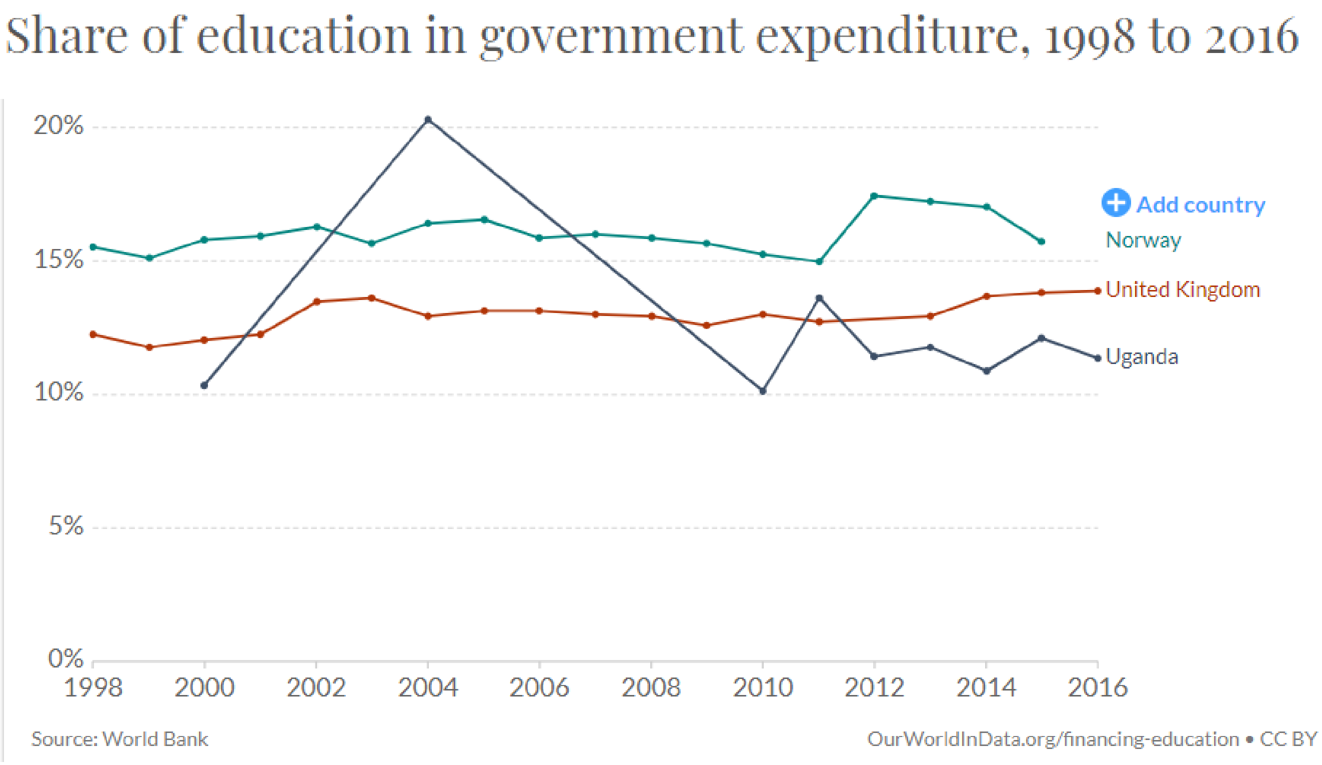
The percentage of pupils in a school passing GCSE maths with a grade 5 or equivalent.

year	2015	2016	2017	2018	2019
% of students passing maths GCSE	27	34	43	37	40



The table and the line graph represent the same data. The graph provides a visual representation of the changes in the school's performance over a period of time, but the points relating to the years would not normally be joined up; they have been here to help to show the trend.

Worked examples



Share of education in government expenditure, 1998 to 2016

Source: World Bank

OurWorldInData.org/financing-education • CC BY

Example 1

By how many percentage points did the share of education in government increase from 2000 to 2016 in the United Kingdom?

Find 2000 on the 'year' axis, move vertically upwards to the red graph line for the United Kingdom, then horizontally across to the 'percentage' axis. The corresponding value is 12 %.

Now do the same for 2016. The value on the 'percentage' axis for 2016 is 14%. Subtract the 2000 figure from the 2016 figure to obtain the percentage point increase: 14%- 12% = 2%.

So, the percentage spent on Education from 2000 to 2016 increased by 2 percentage points.

Example 2

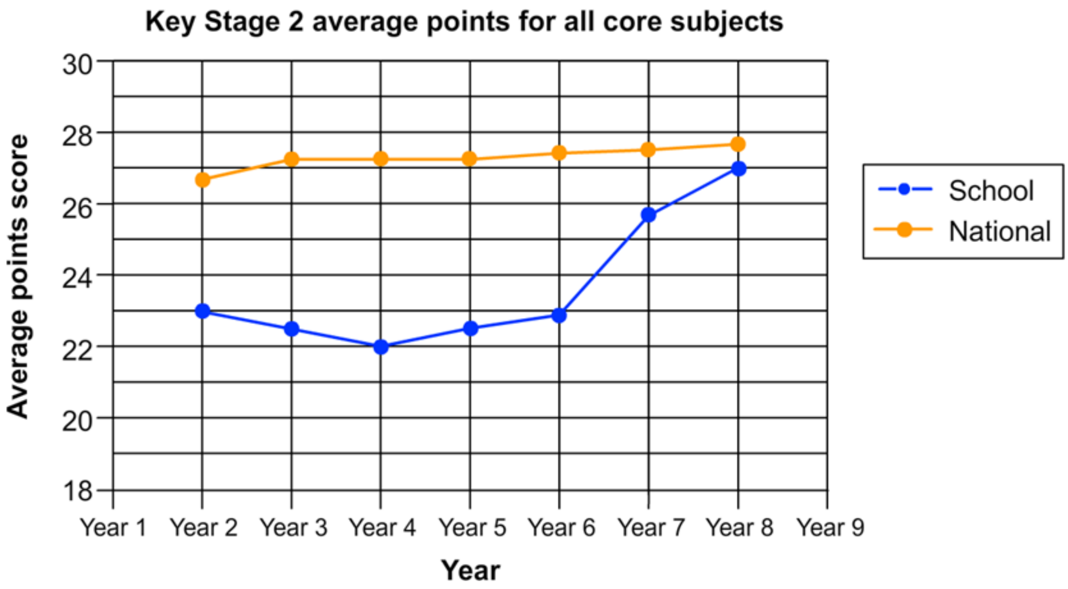
Data was not collected every year between 2000 and 2016 for Uganda. For the years that there is data for Uganda, how many show the Ugandan Governments percentage expenditure on education is higher than the United Kingdoms?

Remembering that the lines are there to show the trend only and are not real data points, we can only use “dots” as these represent the data used to draw the time series graph. The data for Uganda is only available for 2000, 2004, 2010 then yearly to 2016. Using only these years, the Ugandan expenditure line is above the UK’s for 2004 and 2011.

So, for two years the Ugandan Governments percentage expenditure on education higher than the United Kingdoms.

Now your turn 3.5

Primary teachers in the core subjects (English, mathematics and science) attended a meeting to review progress at Key Stage 2 over the last 9 years. The most recent results are labelled as year 9, as its 9th year of the tests.



3.5a

Tick all the true statements:

☐ The school's lowest average points score was in Year 4.

☐ The school's average points score fell from Year 2 to Year 4.

☐ Since Year 4 the trend of improvement in the school's average points score has been greater than the trend of improvement in the national average points score.

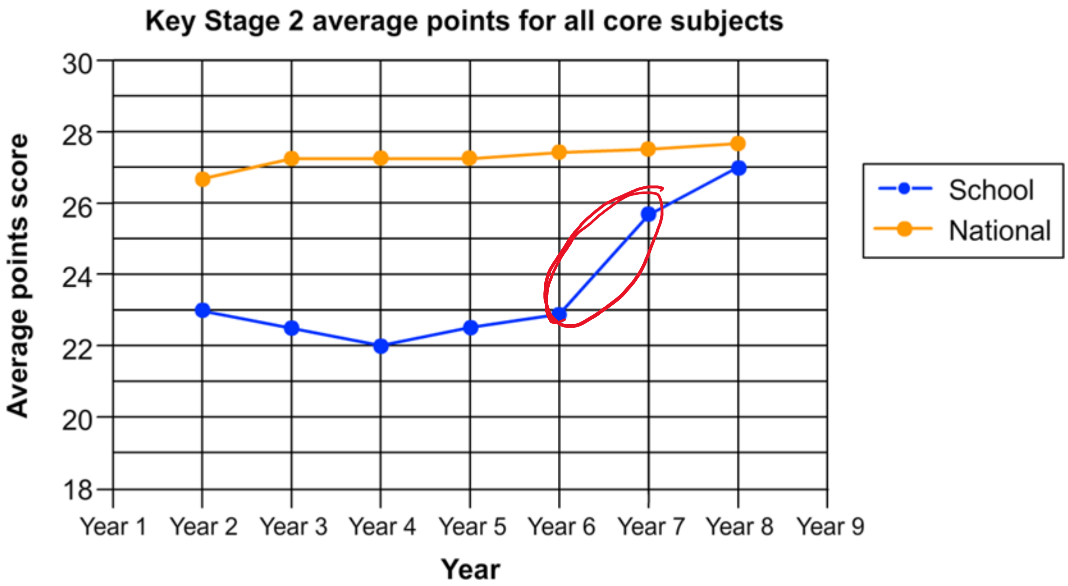
3.5b

On the graph, circle the year in which the school made the greatest progress in average points score.

The answers are on the next page.

Now your turn 3.5 worked solution

Primary teachers in the core subjects (English, mathematics and science) attended a meeting to review progress at Key Stage 2 over the last 9 years. The most recent results are labelled as year 9, as its 9th year of the tests.



3.5a

Tick all the true statements:

☑ The school's lowest average points score was in Year 4.

The blue line is at is lowest in year 4

☑ The school's average points score fell from Year 2 to Year 4.

The blue line is going down, showing scores are decreasing

☑ Since Year 4 the trend of improvement in the school's average points score has been greater than the trend of improvement in the national average points score.

Since year 4 both lines show an increasing trend. The gap between them is getting smaller so the school is improving faster than the National average.

3.5b

On the graph, circle the year in which the school made the greatest progress in average points score.

The steepest upward line is between year 6 and year 7

3.5 Further learning and support.

Time (line) graphs https://corbettmaths.com/2013/05/22/line-graphs/

Glossary of numeracy terms

These terms are used in numeracy. You can use them as part of your preparation for the numeracy professional skills test. You will not be assessed on definitions of terms during the test.

A

Accuracy

The degree of precision given in the question or required in the answer. For example, a length might be measured to the nearest centimetre. A pupil's reading age is usually given to the nearest month, while an average (mean) test result might be rounded to 1 decimal place.

B

Bar chart

A chart where the number associated with each item is shown either as a horizontal or a vertical bar and where the length of the bar is proportional to the number it represents. The length of the bar is used to show the number of times the item occurs, or the value of the item being measured.

Box and whisker diagram

Diagram showing the range and quartile values for a set of data.

C

Cohort

A group having a common quality or characteristic. For example, ‘pupils studying GCSE German this year have achieved higher grades than last year's cohort’ (pupils studying GCSE German last year).

Consistent

Following the same pattern or style over time with little change. For example, a pupil achieved marks of 84, 82, 88 and 85 % in a series of mock GCSE tests; her performance was judged to be consistently at the level needed to obtain GCSE grade A*.

Conversion

The process of exchanging 1 set of units for another. Measurement and currency, for example, can be converted from 1 unit to another, eg centimetres to metres, pounds sterling to euros. Conversion of 1 unit to the other is usually done by using:

- a rule (eg ‘multiply by 5/8 to change kilometres into miles’);
- a formula (eg F= 9/5C + 32, for converting degrees Celsius to degrees Fahrenheit); or
- a conversion graph.

Correlation

The extent to which 2 quantities are related. For example, there is a positive correlation between 2 tests, A and B, if a person with a high mark in test A is likely to have a high mark in test B and a person with a low mark in test A is likely to get a low mark in test B. A scatter graph of the 2 variables may help when considering whether a correlation exists between the 2 variables.

Cumulative frequency graph

A graph in which the frequency of an event is added to the frequency of those that have preceded it. This type of graph is often used to answer a question such as, 'how many pupils are under nine- years-old in a local authority (LA)?' or 'what percentage of pupils gained at least the pass mark of 65 on a test?'

D

Decimal

Numbers based on or counted in a place value system of tens. Normally we talk about decimals when dealing with tenths, hundredths and other decimal fractions less than 1. A decimal point is placed after the unit’s digit in writing a decimal number (eg 1.25). The number of digits to the right of the decimal point up to and including the final non-zero digit is expressed as the number of decimal places. In the example above there are 2 digits after the decimal point, and the number is said to have 2 decimal places, sometimes expressed as 2 dp. Many simple fractions cannot be expressed exactly as a decimal. For example, the fraction 1/3 as a decimal is 0.3333, which is usually represented as 0.3 recurring. Decimals are usually rounded to a specified degree of accuracy (eg 0.6778 is 0.68 when rounded to 2 dp. 0.5 is always rounded up, so 0.5 to the nearest whole number is 1).

Distribution

The spread of a set of statistical information. For example, the number of absentees on a given day in a school is distributed as follows:

Monday: 5 Tuesday; 17 Wednesday: 23 Thursday: 12 Friday: 3

A distribution can also be displayed graphically.

F

Formula

A relationship between numbers or quantities expressed using a rule or an equation. For example, final mark = (0.6 x mark 1) + (0.4 x mark 2).

Fraction

Fractions are used to express parts of a whole (eg three quarters). The number below the division line, the denominator, records the number of equal parts into which the number above the division line, the numerator, has been divided.

Frequency

The number of times an event or quantity occurs.

G

Greater than

A comparison between 2 quantities. The symbol > is used to represent 'greater than', eg 7>2, or
>5%.

I

Interquartile range

The numerical difference between the upper quartile and the lower quartile. The lower quartile of a set of data has one quarter of the data below it and three quarters above it. The upper quartile has three quarters of the data below it and one quarter above it. The interquartile range represents the middle 50% of the data.

L

Line graphs

A graph on which the plotted points are joined by a line. It is a visual representation of 2 sets of related data. The line may be straight or curved. It is often used to show a trend, such as how a particular value is changing over time.

M

Mean

One measure of the 'average' of a set of data. The 'mean' average is usually used when the data involved is fairly evenly spread. For example, the individual costs of 4 textbooks are £9.95, £8.34, £11.65 and £10.50. The mean cost of a textbook is found by totalling the 4 amounts, giving £40.44 and then dividing by 4, which gives £10.11. The word ‘average’ is frequently used in place of the mean, but this can be confusing as both median and mode are also ways of expressing an average.

Median

Another measure of the 'average' of a set of data. It is the middle number of a series of numbers or quantities when arranged in order, eg from smallest to largest. For example, in the following series of numbers the median is 7:

2, 4, 5, 7, 8, 15 and 18

When there is an even number of numbers, the median is found by adding the 2 middle numbers and then halving them. For example, in the following series of numbers the median is (23 + 30) / 2 = 26.5.

12, 15, 23, 30, 31 and 45

Median and quartile lines

Quartiles can be found by taking a set of data that has been arranged in increasing order and dividing it into 4 equal parts. The first quartile is the value of the data at the end of the first quarter. The median quartile is the value of the data at the end of the second quarter. The third quartile is the value of the data at the end of the third quarter. Quartile lines can be used to show pupils' progression from 1 key stage to another, when compared with national or local data.

Mode

Another measure of the 'average' of a set of data. It is the most frequently occurring result in any group of data. For example, in the following set of exam results the mode is 31% because this value appears most frequently:

30%, 34%, 36% 31%, 31%, 30%, 34%, 33%, 31% and 32%

O

Operations

The means of combining 2 numbers or sets of numbers. For example, addition, subtraction, multiplication and division.

P

Percentage

A fraction with a denominator of 100, but written as the numerator followed by '%', eg 30 over 100 or 30%. A fraction that is not in hundredths can be converted so that the denominator is 100, eg 65 over 100 = 65%. Percentages can be used to compare different fractional quantities. For example, in class A, 10 pupils out of 25 are studying French; in class B, 12 out of 30 pupils are studying French. However, both 10/25 and 12/30 are equivalent to 4/10, or 40%. The same percentage of pupils, therefore, studies French in both these classes.

Percentage points

The difference between 2 values, given as percentages. For example, a school has 80% attendance one year and 83% the next year. There has been an increase of 3 percentage points in attendance.

Percentile

The values of a set of data that has been arranged in order and divided into 100 equal parts. For example, a year group took a test and the 60th percentile was at a mark of 71. This means that 60% of the cohort scored 71 marks or less. The 25th percentile is the value of the data such that 25% or one quarter of the data is below it and so is the same as the lower quartile. Similarly, the 75th percentile is the same as the upper quartile and the median is the same as the 50th percentile.

Pie chart

A pie chart represents the 360° of a circle and is divided into sectors by straight lines from its centre to its circumference. Each sector angle represents a specific proportion of the whole. Pie charts are used to display the relationship of each type or class of data within a whole set of data in a visual form.

Prediction

A statement based on analysing statistical information about the likelihood that a particular event will occur. For example, an analysis of a school's exam results shows that the number of pupils achieving grades 4+ in science at a school has increased by 3% per year over the past 3 years. On the basis of this information the school predicts that the percentage of pupils achieving grades 4+ in science at the school next year will increase by at least 2%.

Proportion

A relationship between 2 values or measures. These 2 values or measures represent the relationship between some part of a whole and the whole itself. For example, a year group of 100 pupils contains 60 boys and 40 girls, so the proportion of boys in the school is 60 out of 100 or 3 out of 5. This is usually expressed as a fraction, in this case, three fifths.

Q

Quartile (lower)

The value of a set of data at the first quarter, 25%, when all the data has been arranged in ascending order. It is the median value of the lower half of all the values in the data set. For example, the results of a test were: 1, 3, 5, 6, 7, 9, 11, 15, 18, 21, 23 and 25. The median is 10. The
values in the lower half are 1, 3, 5, 6, 7 and 9. The lower quartile is 5.5. This means that one quarter of the cohort scored 5.5 or less. The lower quartile is also the 25th percentile.

Quartile (upper)

The value of a set of data at the third quarter, 75%, when that data has been arranged in ascending order. It is the median value of the upper half of all the values in the data set. In the lower quartile example, the upper quartile is 19.5, the median value of the upper half of the data set. Three- quarters of the marks lie below it. The upper quartile is also the 75th percentile.

R

Range

The difference between the lowest and the highest values in a set of data. For example, for the set of data 12, 15, 23, 30, 31 and 45, the range is the difference between 12 and 45. 12 is subtracted from 45 to give a range of 33.

Ratio

A comparison between 2 numbers or quantities. A ratio is usually expressed in whole numbers. For example, a class consists of 12 boys and 14 girls. The ratio of boys to girls is 12:14. This ratio may be described more simply as 6:7 by dividing both numbers by 2. The ratio of girls to boys is 14:12 or 7:6.

Rounding

Expressing a number to a degree of accuracy. This is often done in contexts where absolute accuracy is not required, or not possible. For example, it may be acceptable in a report to give outcomes to the nearest 100 or 10. So the number 674 could be rounded up to 700 to the nearest 100, or down to 670 to the nearest 10. If a number is half way or more between rounding points, it is conventional to round it up, eg 55 is rounded up to 60 to the nearest 10 and 3.7 is rounded up to 4 to the nearest whole number. If the number is less than half way, it is conventional to round down, eg 16.43 is rounded down to 16.4 to 1 decimal place.

S

Scatter graph

A graph on which data relating to 2 variables is plotted as points, each axis representing 1 of the variables. The resulting pattern of points indicates how the 2 variables are related to each other. This type of graph is often used to demonstrate or confirm the presence or absence of a correlation between the 2 variables, and to judge the strength of that correlation.

Sector

The part or area of a circle which is formed between 2 radii and the circumference. Each piece of a pie chart is a sector.

Standardised scores

Standardised scores are used to enable comparisons on tests between different groups of pupils. Tests are standardised so that the average national standardised scores automatically appears as 100, so it is easy to see whether a pupil is above or below the national average.

T

Trend

The tendency of data to follow a pattern or direction. For example, the trend of the sequence of numbers 4, 7, 11, 13 and 16 is described as ‘increasing’.

V

Value-added

The relationship between a pupil's previous attainment and their current attainment gives a measure of their progress. Comparing this with the progress made by other pupils gives an impression of the value added by a school.

Variables

The name given to a quantity which can take any 1 of a given set of values. For example, on a line graph showing distance against time, both distance and time are variables. A variable can also be a symbol which stands for an unknown number, and that can take on different values. For example, the final mark in a test is obtained by a formula using the variables A and B as follows: final mark = (Topic 1 mark x A) + (Topic 2 mark x B).

W

Weighting

A means of attributing relative importance to 1 or more of a set of results. Each item of data is multiplied by a pre-determined amount to give extra weight to 1 or more components. For example, marks gained in year 3 of a course may be weighted twice as heavily as those gained in the first 2 years, in which case those marks would be multiplied by 2 before finding the total mark for the course.

Whole number

A positive integer (eg 1, 2, 3, 4, 5).

FUNDAMENTAL MATHEMATICS PRACTICE TEST

Name:

School:

To be completed in examination conditions. The pass mark is 11 fully correct answers to any of the 15 questions.

The test has two sections. Section A has 7 questions and Section B has 8 questions.

All questions should be attempted.

Section A needs to be completed before starting section B.

The total time limit for both sections combined is 40 minutes.

It is recommended that you spend approximately 15 minutes on section A and the rest on the longer section B.

Tes Institute School Direct Programme

Fundamental Mathematics Practice Test

Section A

Non-Calculator skills

Answer the following questions without a calculator. Pen and paper workings are allowed and should be shown with your answers.

1. In a school there are eight year 7 classes. Six of these have 29 students, one has 30 and the other has 31. In In total, how many students are in the year group?

…………………… students

(FS 1.1)

2. All 30 students in a class took part in a sponsored walk around the school field for charity. The students were expected to walk on average 16 laps. The average amount of sponsorship is 51p per lap. Calculate the expected total amount of money that will be raised for the charity.

£ …………………………..

(FS 1.1 and FS 2.7)

3. In a recent Science test, a student had a mark of 135. Analysis of previous data suggests that on the next test the same student should achieve 20% higher. What score should this student achieve?

…………. marks

(FS 1.2)

4. A school trip costs £320, of which ¾ is the cost of hiring two coaches. How much does **EACH coach** cost to hire?

£ …………….……

(FS 1.2)

5. A school is looking at changing its timings of the school day. One proposal is to have a 15-minute tutor period at 0825, followed by two lessons of 55 minutes each, a break of 20 minutes and two further 55-minute lessons before a lunch break. In this proposal, what time will the lunch break be?

lunchtime ……………………

(FS 1.1)

6. A year 10 Art teacher is planning his resource needs for the academic year. Each of his 26 students will need sketch books. A pack of 10 costs £12.59. His budget for these is £84.
   a) **Estimate** the total number of sketch books he can purchase.
   b) **Approximately** how many sketch books will each student get?

Stapled Sketchbooks - A4 Pack of ten

Product code: HE1826083

Product Description >

£12.59

£15.11 inc VAT

In stock ●

1

nd-pads/sketchbooks-and-pads/stapled-sketchbooks-a4/he1826083

………………….. total books

……………………….. books per student

(FS 1.3)

Section B

Mathematical calculations, problem solving and data interpretation

In the following questions, a calculator is permitted and recommended. However, you are encouraged to record your methods with each question.

7. Twelve out of a year 5 class of 28 pupils attend the after-school club. What percentage of the class attend the after-school club? Give your answer to one decimal place.

……………… %

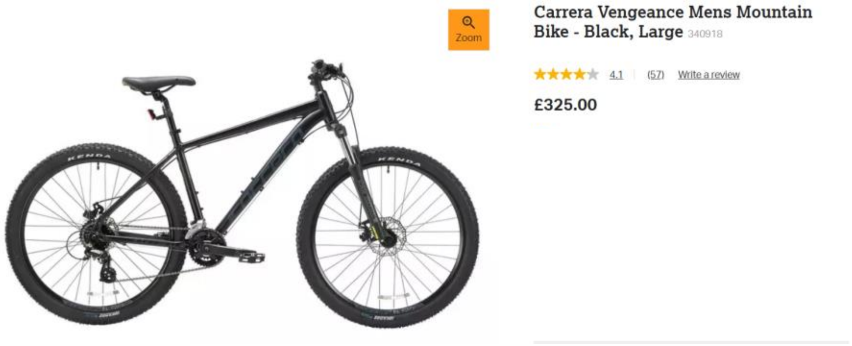
(FS 2.1)

8. In the Academic Year 2017/18 approximately 8.7% of all primary school aged enrolled pupils in the England were classified as having persistent absence. If during this time, there were 3 954 310 pupils enrolled, how many were classified as having persistent absence.

………………………….. pupils

(FS 2.2)

9. The website https://www.discountsforteachers.co.uk/ offers a 7.5% discount at Halfords for its registered members. What would a member pay for the bike shown below?



Carrera Vengeance Mens Mountain Bike - Black, Large 340918

★★★★☆ 4.1 | (57) Write a review

£325.00

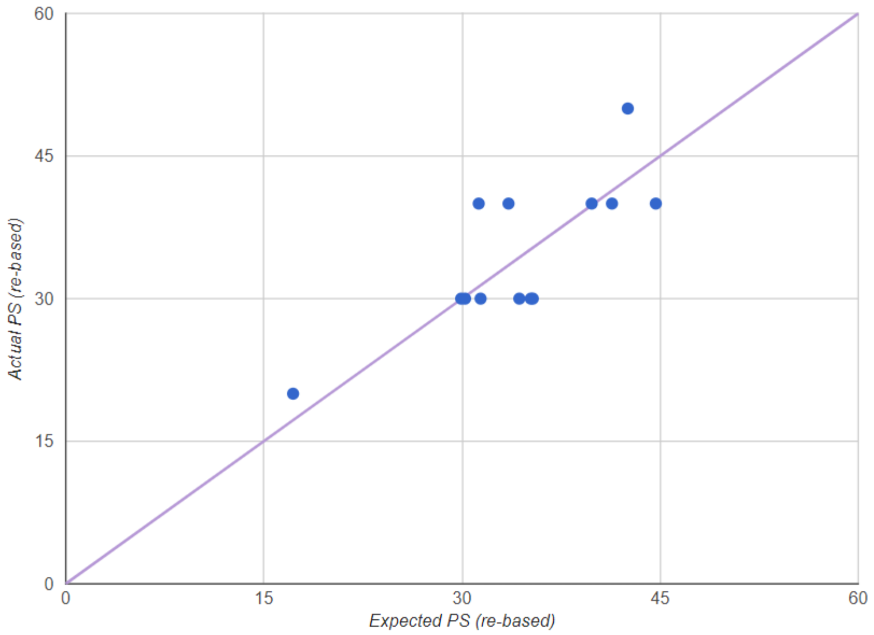
Source: https://www.halfords.com/bikes/mountain-bikes/carrera-vengeance-mens-mountain-bike-2020---black---xs-s-m-l-xl-frames-340910.html

£ ……………………

(FS 2.3)

10. A pupil obtained the following marks in three exam papers.
The diagram below shows expected performance against actual performance for a small English class.

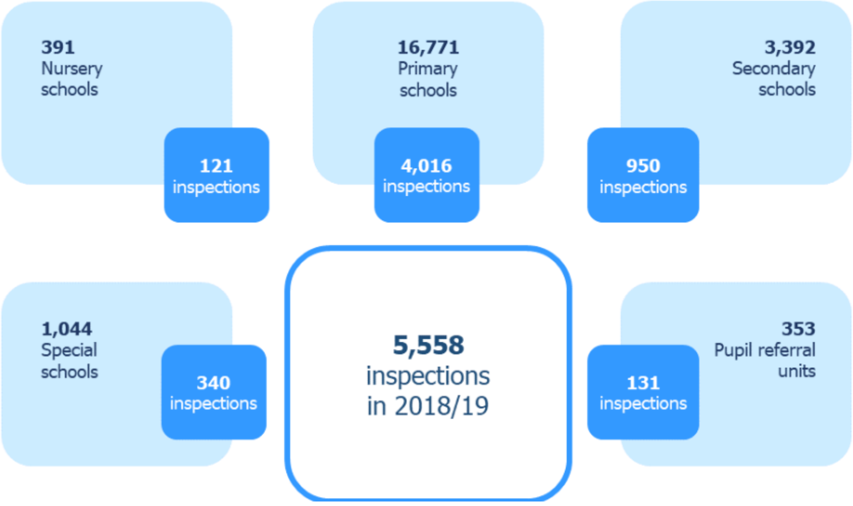
How many students performed better than expected?



………………………… Students

(FS 3.3)

11. The figure below shows the OFSTED inspections carried out in 2018/19 and the number of schools as of August 2019



https://www.gov.uk/government/publications/state-funded-schools-inspections-and-outcomes-as-at-31-august-2019/state-funded-schools-inspections-and-outcomes-as-at-31-august-2019-main-findings

Tick **all** the **true** statements

☐ The percentage of nursery schools inspected was 35.9%.

☐ The ratio of special schools inspected to secondary schools inspected, expressed in its simplest form is 34:95.

☐ The school type with the largest proportion being inspected was Pupil referral units.

☐ Approximately 25% of all types of school were inspected in the academic year 2018/19.

(FS 2.4, 2.5 and 2.6)

12. For a school visit to Hong Kong, each of the 28 students were allowed to take £75 spending money. Prior to the visit, the teacher collected the money and exchanged it for Hong Kong Dollars.

✓ Your best exchange rate

1GBP = 9.9458 HKD

The exchange rate was £1.00 = 9.9458 HKD

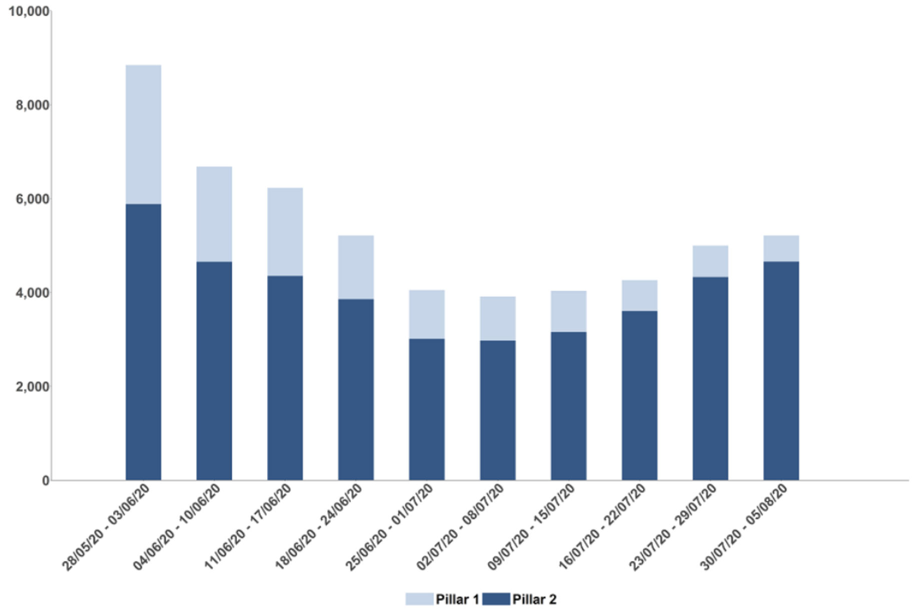
How many Hong Kong dollars did the teacher receive?

………………………… HKD

(FS 2.7)

13. The figure below shows the number of people newly testing positive for COVID 19 in England over a 10-week period. Pillar 1 tests are conducted in Hospitals and Pillar 2 are conducted within the community.

Figure 3: number of people newly testing positive for COVID-19 by pillar, England



`Source: https://www.gov.uk/government/publications/nhs-test-and-trace-england-and-coronavirus-testing-uk-statistics-30-july-to-5-august-2020/weekly-statistics-for-nhs-test-and-trace-england-and-coronavirus-testing-uk-30-july-to-5-august

Tick the **FALSE** statement.

☐ The number of people newly testing positive has both decreased and increased over the 10 weeks

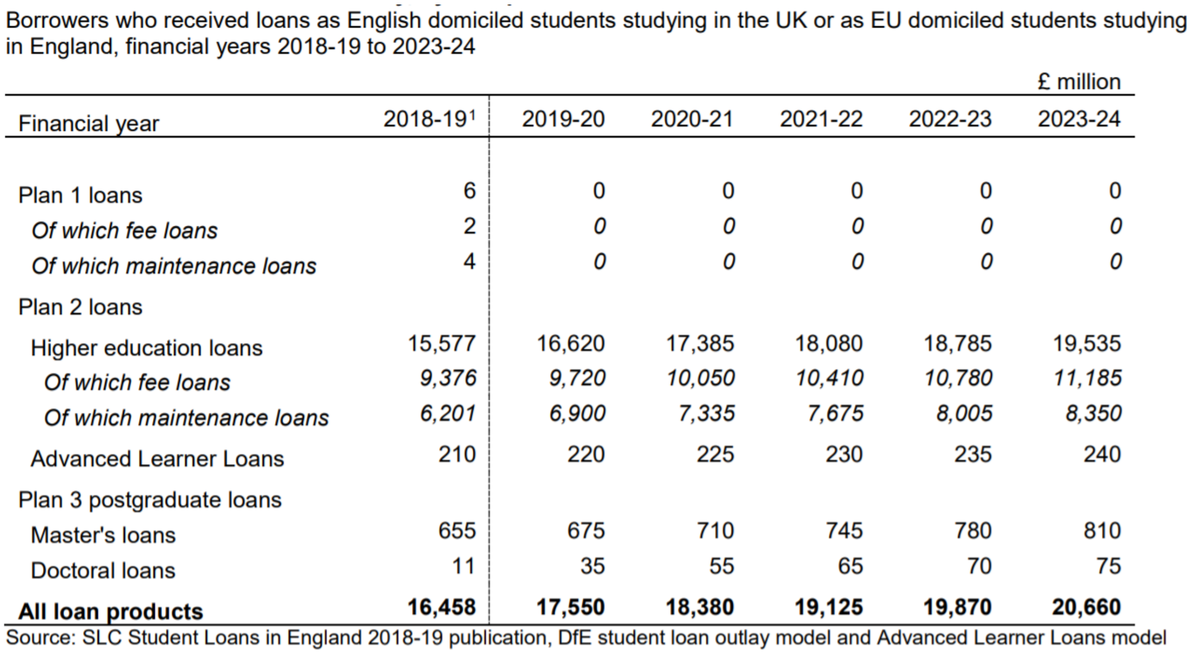
☐ Each week of the time week period, over half of people newly testing positive are in the community.

☐ The number of people newly testing positive has increased since the week beginning 25/06/20

(FS 3.2 and 3.5)

14. The table below shows information about student loan forecasts.

Calculate the **mean** for the “All loan products” forecasts for the 5-year period 2019-20 to 2023-24



Borrowers who received loans as English domiciled students studying in the UK or as EU domiciled students studying in England, financial years 2018-19 to 2023-24

£ million

Financial year	2018-19[1]	2019-20	2020-21	2021-22	2022-23	2023-24
Plan 1 loans	6	0	0	0	0	0
Of which fee loans	*2*	*0*	*0*	*0*	*0*	*0*
Of which maintenance loans	*4*	*0*	*0*	*0*	*0*	*0*
Plan 2 loans						
Higher education loans	15,577	16,620	17,385	18,080	18,785	19,535
Of which fee loans	*9,376*	*9,720*	*10,050*	*10,410*	*10,780*	*11,185*
Of which maintenance loans	*6,201*	*6,900*	*7,335*	*7,675*	*8,005*	*8,350*
Advanced Learner Loans	210	220	225	230	235	240
Plan 3 postgraduate loans						
Master's loans	655	675	710	745	780	810
Doctoral loans	11	35	55	65	70	75
All loan products	**16,458**	**17,550**	**18,380**	**19,125**	**19,870**	**20,660**

Source: SLC Student Loans in England 2018-19 publication, DfE student loan outlay model and Advanced Learner Loans model

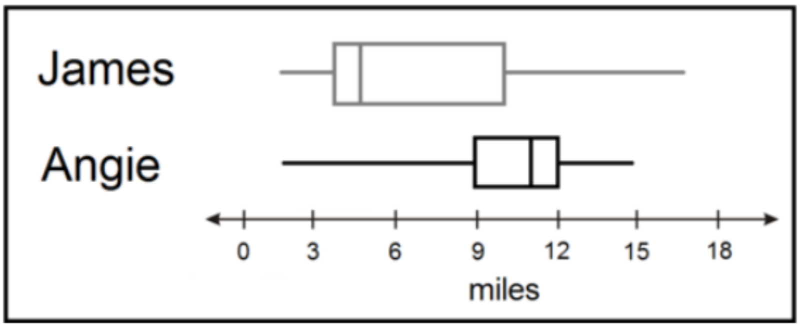
https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/811997/Student_loan_forecasts_2018-19_-_text.pdf

£ ………………………….millions

(FS 3.1)

15. Two trainee teachers have taken up running to keep fit. The box plot shows the number of miles per day that James and Angie ran in a month.

Which trainee teacher ran a more varied number of miles per day?



……………………………

(FS 3.4)

FUNDAMENTAL MATHEMATICS PRACTICE TEST

Name: Worked Solutions

School:

To be completed in examination conditions. The pass mark is 11 fully correct answers to any of the 15 questions.

The test has two sections. Section A has 7 questions and Section B has 8 questions.

All questions should be attempted.

Section A needs to be completed before starting section B.

The total time limit for both sections combined is 40 minutes.

It is recommended that you spend approximately 15 minutes on section A and the rest on the longer section B.

Tes Institute School Direct Programme

Fundamental Mathematics Practice Test

Section A

Non-Calculator skills

Answer the following questions without a calculator. Pen and paper workings are allowed and should be shown with your answers.

Any non calculator method that obtains the correct answer is acceptable. These may be the most efficient.

1. In a school there are eight year 7 classes. Six of these have 29 students, one has 30 and the other has 31. In In total, how many students are in the year group?

$6 \times 29 + 30 + 31$

$$\begin{aligned} 6 \times (30-1) &= 6 \times 30 - 6 \times 1 \\ &= 180 - 6 \\ &= 174 \end{aligned}$$

$30 + 31 = 61$

$$\begin{array}{r} 174 \\ +\ 61 \\ \hline 235 \end{array}$$

235 students

(FS 1.1)

2. All 30 students in a class took part in a sponsored walk around the school field for charity. The students were expected to walk on average 16 laps. The average amount of sponsorship is 51p per lap. Calculate the expected total amount of money that will be raised for the charity.

$30 \times 16 \times 0.51$

$$\begin{aligned} 16 \times 30 &= 16 \times 3 \times 10 \\ &= 48 \times 10 \\ &= 480 \text{ total laps} \end{aligned}$$

$$\begin{aligned} 480 \times 0.51 &= 480 \times 0.5 + 480 \times 0.01 \\ &= 240 + 4.8 \end{aligned}$$

$$\begin{array}{r} 240 \\ +\ 4.8 \\ \hline 244.8 \end{array}$$

£ 244.80

(FS 1.1 and FS 2.7)

3. In a recent Science test, a student had a mark of 135. Analysis of previous data suggests that on the next test the same student should achieve 20% higher. What score should this student achieve?

10% of 135 = 135 ÷ 10 = 13.5

20% of 135 = 13.5 × 2 = 27

New mark =
$$\begin{array}{r} 135 \\ +\ 27 \\ \hline 162 \\ 1 \end{array}$$

162 marks

(FS 1.2)

4. A school trip costs £320, of which ¾ is the cost of hiring two coaches. How much does **EACH coach** cost to hire?

$\frac{3}{4}$ of £320 = 320 ÷ 4 × 3

= 320 ÷ 2 ÷ 2 × 3

= 80 × 3

two coaches = £240

240 ÷ 2 = £120 per coach

£ 120

(FS 1.2)

5. A school is looking at changing its timings of the school day. One proposal is to have a 15-minute tutor period at 0825, followed by two lessons of 55 minutes each, a break of 20 minutes and two further 55-minute lessons before a lunch break. In this proposal, what time will the lunch break be?

(4 lessons of 55 mins + break 20 mins) + 15 min tutor time

4 × 5 mins

add to 4 × 55 mins

is 4 hours.

↓

4 hours + 15 mins

0825 + 4 hours = 1225

1225 + 15 mins = 1240

lunchtime 12:40

(FS 1.1)

6. A year 10 Art teacher is planning his resource needs for the academic year. Each of his 26 students will need sketch books. A pack of 10 costs £12.59. His budget for these is £84.
   c) **Estimate** the total number of sketch books he can purchase.
   d) **Approximately** how many sketch books will each student get?

Stapled Sketchbooks - A4 Pack of ten

Product code: HE1826083

Product Description >

£12.59

£15.11 inc VAT

In stock

Source: https://www.hope-education.co.uk/product/art-and-design/art-books-and-pads/sketchbooks-and-pads/stapled-sketchbooks-a4/he1826083

a) Calculation: $84 \div £12.59 \times 10$

Estimation $= 84 \div 12 \times 10$ OR $80 \div 10 \times 10 = 80$

$= 7 \times 10 = 70$

Practically if $84 \div 12 = 7$, he can only actually buy 6 packs, 60 books so this is also acceptable

60, 70 or 80 total books

2 or 3 books per student

(FS 1.3)

b) $60 \div 25 \approx 2$ each $80 \div 25 \approx 3$ each or 2 each with some left

$70 \div 25 \approx 3$ each

Section B

Mathematical calculations, problem solving and data interpretation

In the following questions, a calculator is permitted and recommended. However, you are encouraged to record your methods with each question.

7. Twelve out of a year 5 class of 28 pupils attend the after-school club. What percentage of the class attend the after-school club? Give your answer to one decimal place.

$12 \div 28 \times 100 = 0{\cdot}42857\ldots \times 100$

$= 42{\cdot}857\ldots$

$= 42{\cdot}9$

42·9 %

(FS 2.1)

8. In the Academic Year 2017/18 approximately 8.7% of all primary school aged enrolled pupils in the England were classified as having persistent absence. If during this time, there were 3 954 310 pupils enrolled, how many were classified as having persistent absence.

$8{\cdot}7\%$ of $3\,954\,310 = 8{\cdot}7 \div 100 \times 3\,954\,310$

$= 0{\cdot}087 \times 3\,954\,310$

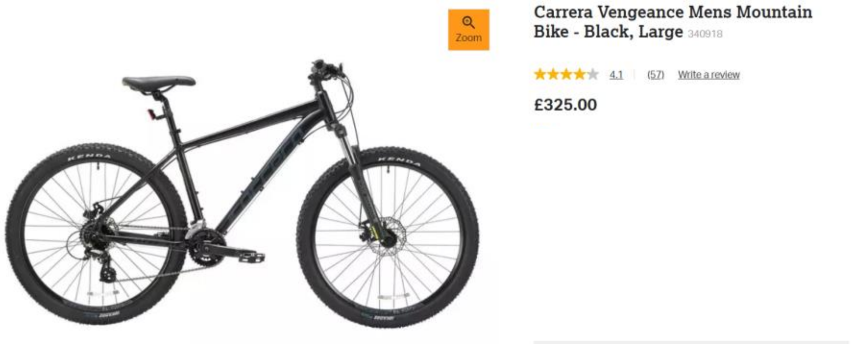
$= 344024{\cdot}97$

≈ 344025

344024 or 344025 pupils

(FS 2.2)

9. The website https://www.discountsforteachers.co.uk/ offers a 7.5% discount at Halfords for its registered members. What would a member pay for the bike shown below?



Carrera Vengeance Mens Mountain Bike - Black, Large 340918

★★★★☆ 4.1 | (57) Write a review

£325.00

Source: https://www.halfords.com/bikes/mountain-bikes/carrera-vengeance-mens-mountain-bike-2020---black---xs-s-m-l-xl-frames-340910.html

100 − 7.5 = 92.5%

92.5% of £325 = 0.925 × £325

= £300.625

= £300.63

Discount = 7.5% of £325

= 0.075 × 325

= £24.375

Reduce price = £325 − 24.375

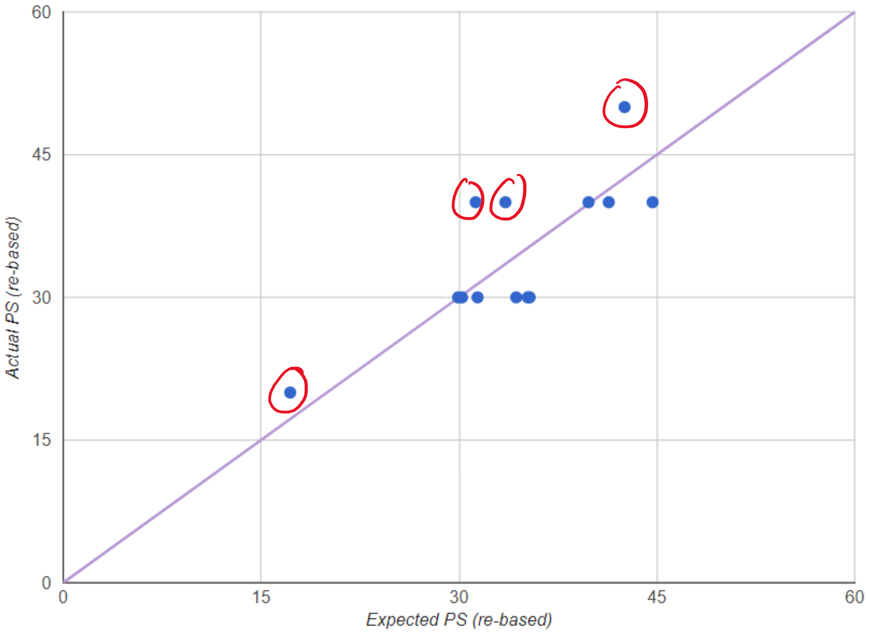
= £300.625

£ 300.62 or 300.63

(FS 2.3)

10. A pupil obtained the following marks in three exam papers.
The diagram below shows expected performance against actual performance for a small English class.

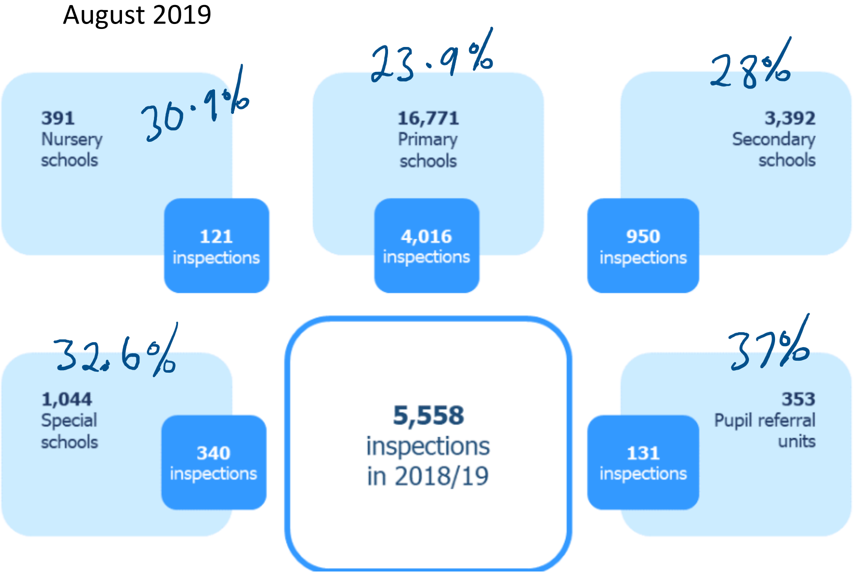
How many students performed better than expected?



4 Students

(FS 3.3)

11. The figure below shows the OFSTED inspections carried out in 2018/19 and the number of schools as of August 2019



The percentages have been calculated to compare the proportion of each school type being inspected. The Nursery schools calculation is shown, the others calculated the same way.

https://www.gov.uk/government/publications/state-funded-schools-inspections-and-outcomes-as-at-31-august-2019/state-funded-schools-inspections-and-outcomes-as-at-31-august-2019-main-findings

Tick **all** the **true** statements

121 ÷ 391 x100 = 30.946....
False

☐ The percentage of nursery schools inspected was 35.9%.

☑ The ratio of special schools inspected to secondary schools inspected, expressed in its simplest form is 34:95.

340 : 950 = 34 : 95
÷10 ÷10

☑ The school type with the largest proportion being inspected was Pupil referral units.

☑ Approximately 25% of all types of school were inspected in the academic year 2018/19.

total schools = 391 + 16771 + 3392 + 1044 + 353
=

(FS 2.4, 2.5 and 2.6)

% inspected = 5558 ÷ 21951 x100 = 25.32... %

12. For a school visit to Hong Kong, each of the 28 students were allowed to take £75 spending money. Prior to the visit, the teacher collected the money and exchanged it for Hong Kong Dollars.


✓ Your best exchange rate

1GBP = 9.9458 HKD

The exchange rate was £1.00 = 9.9458 HKD

How many Hong Kong dollars did the teacher receive?

Per student: £75 × 9.9458 = 745.935

= 745.94 HKD

total = 28 × 745.94 HKD

= 20886.32

other methods will give also get the correct answer.

20886.18

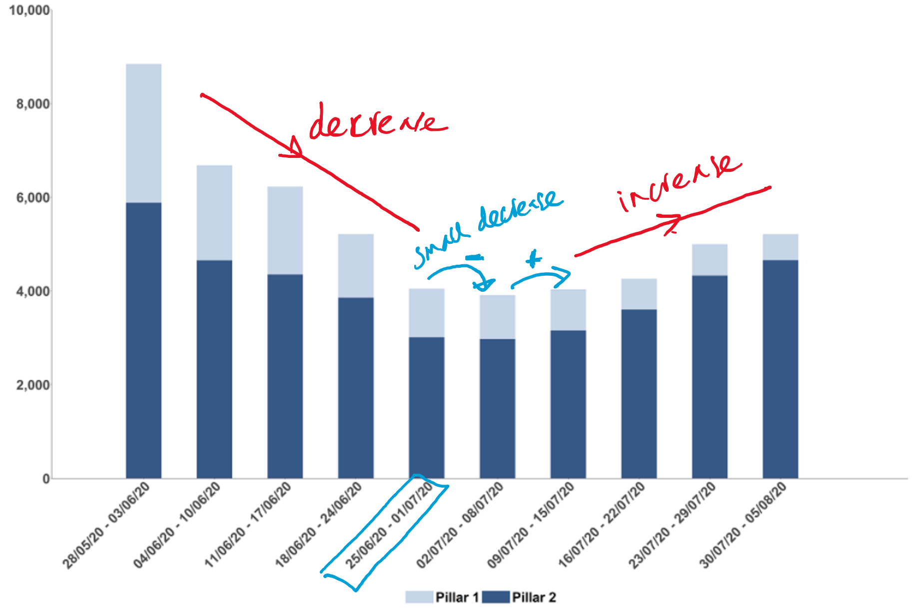
OR

20886.32 .... HKD

(FS 2.7)

13. The figure below shows the number of people newly testing positive for COVID 19 in England over a 10-week period. Pillar 1 tests are conducted in Hospitals and Pillar 2 are conducted within the community.

Figure 3: number of people newly testing positive for COVID-19 by pillar, England



For each bar the dark blue section is larger than the pale blue section. So over $\frac{1}{2}$ is dark blue

`Source: https://www.gov.uk/government/publications/nhs-test-and-trace-england-and-coronavirus-testing-uk-statistics-30-july-to-5-august-2020/weekly-statistics-for-nhs-test-and-trace-england-and-coronavirus-testing-uk-30-july-to-5-august

Tick the **FALSE** statement.

True

☐ The number of people newly testing positive has both decreased and increased over the 10 weeks

True

☐ Each week of the time week period, over half of people newly testing positive are in the community.

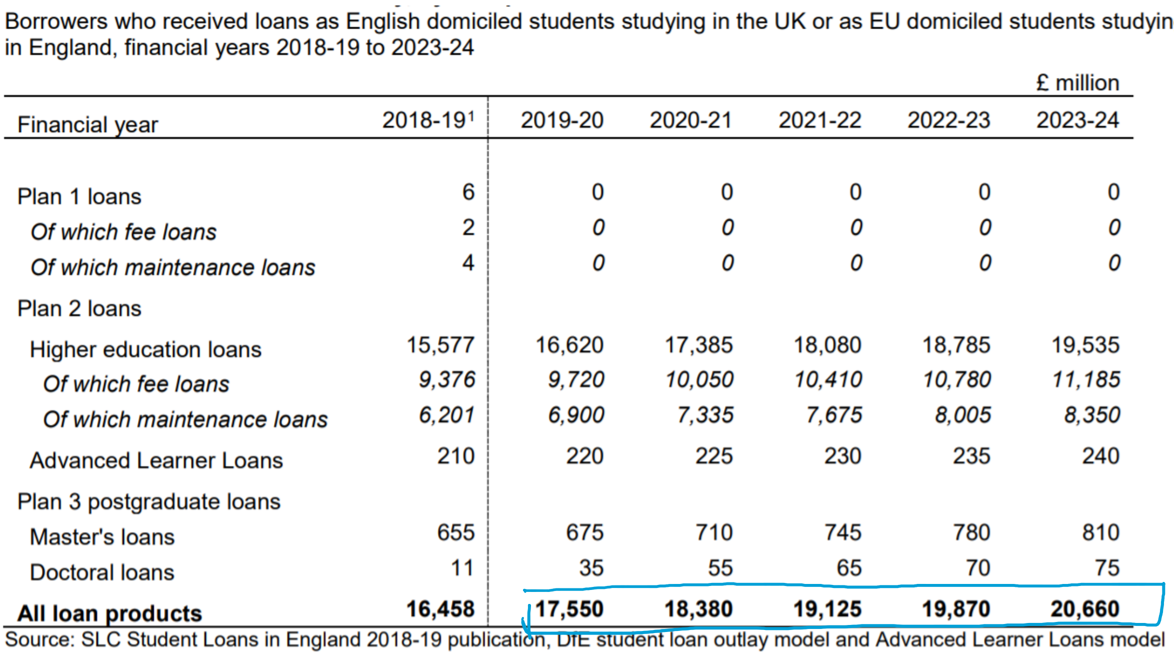
☑ The number of people newly testing positive has increased since the week beginning 25/06/20

There is a small decrease to the week beginning 02/07/20, so not increased from this week.

(FS 3.2 and 3.5)

14. The table below shows information about student loan forecasts.

Calculate the **mean** for the "All loan products" forecasts for the 5-year period 2019-20 to 2023-24



Borrowers who received loans as English domiciled students studying in the UK or as EU domiciled students studying in England, financial years 2018-19 to 2023-24

£ million

Financial year	2018-19[1]	2019-20	2020-21	2021-22	2022-23	2023-24
Plan 1 loans	6	0	0	0	0	0
Of which fee loans	*2*	*0*	*0*	*0*	*0*	*0*
Of which maintenance loans	*4*	*0*	*0*	*0*	*0*	*0*
Plan 2 loans						
Higher education loans	15,577	16,620	17,385	18,080	18,785	19,535
Of which fee loans	*9,376*	*9,720*	*10,050*	*10,410*	*10,780*	*11,185*
Of which maintenance loans	*6,201*	*6,900*	*7,335*	*7,675*	*8,005*	*8,350*
Advanced Learner Loans	210	220	225	230	235	240
Plan 3 postgraduate loans						
Master's loans	655	675	710	745	780	810
Doctoral loans	11	35	55	65	70	75
All loan products	**16,458**	**17,550**	**18,380**	**19,125**	**19,870**	**20,660**

Source: SLC Student Loans in England 2018-19 publication, DfE student loan outlay model and Advanced Learner Loans model

https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/811997/Student_loan_forecasts_2018-19_-_text.pdf

mean = (17550 + 18380 + 19125 + 19870 + 20660) ÷ 5

= 95585 ÷ 5 = 19117

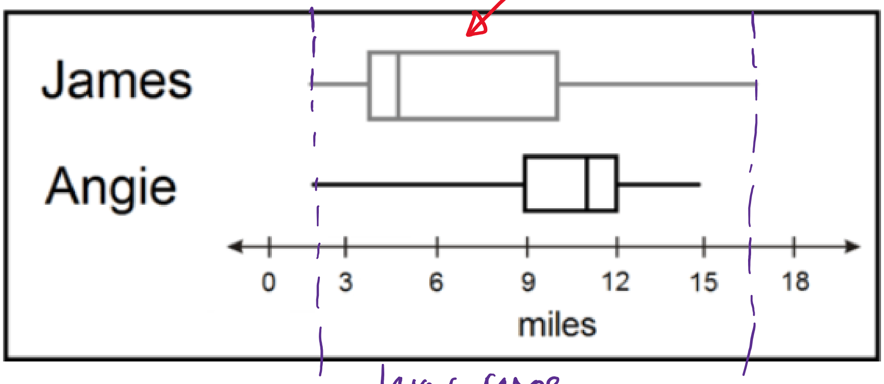
£ ....19 117.......millions

(FS 3.1)

15. Two trainee teachers have taken up running to keep fit. The box plot shows the number of miles per day that James and Angie ran in a month.

Which trainee teacher ran a more varied number of miles per day?

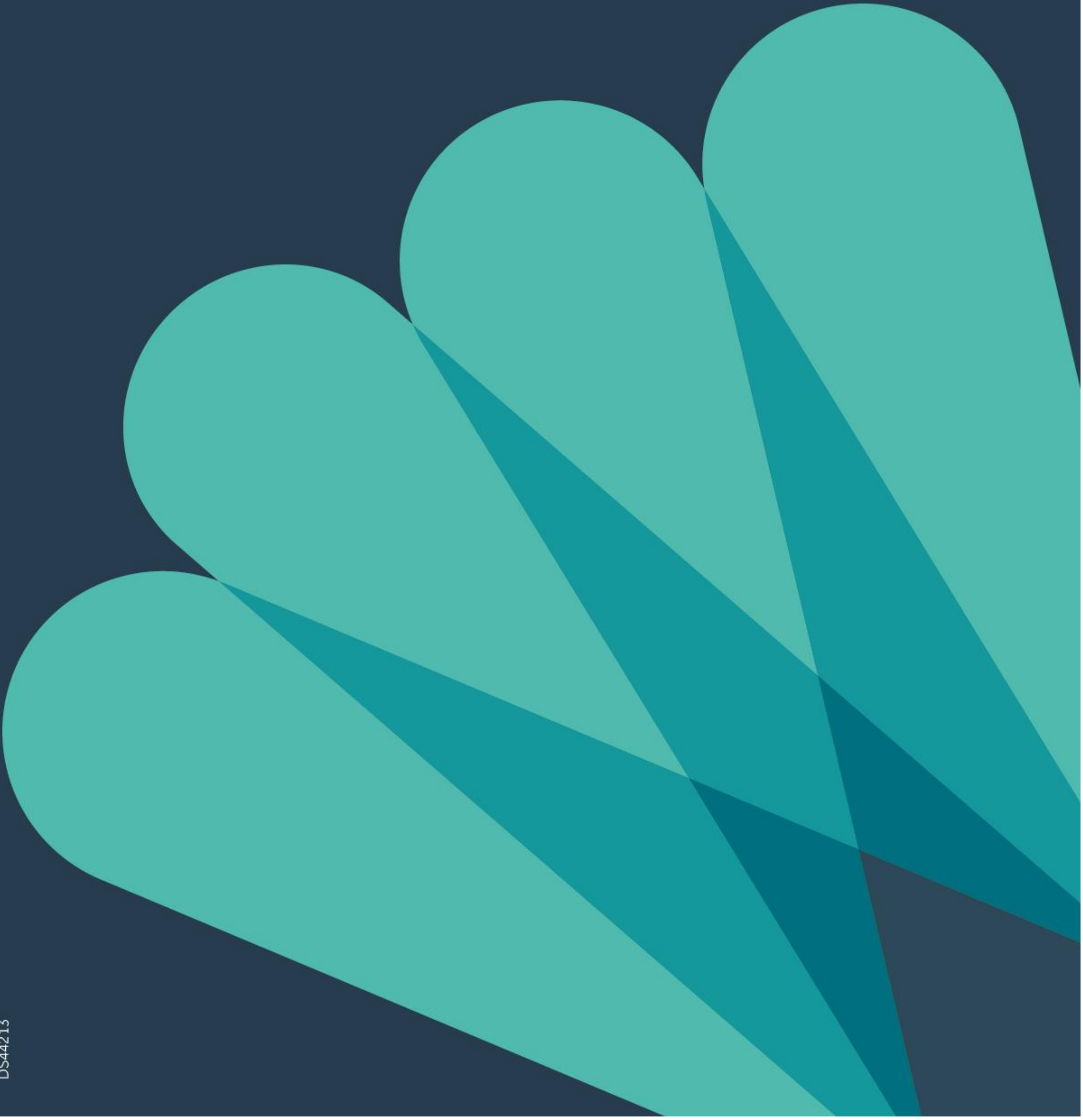
bigger box show larger IQR so more spread.



larger range

.......James.......

(FS 3.4)



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